Problem 1: Interactions between Two Surfaces with Arbitrary Potential. Consider two semi-infinite blocks of the same material separated by a distance h. Compute the interaction energy and force between the two blocks as a function of h when:

(a) The pairwise intermolecular potential between molecules composing the blocks is $V(r) = -Cr^{-m}$, where C is a constant.

(b) The intermolecular potential is $V(r) = -Ce^{-\beta r}$, where β is a positive constant.

Problem 2: Van der Waals Interactions between Two Surfaces with a Finite Thickness. Consider two infinite plates of the same material of thickness d separated by a distance h. The molecules composing the blocks interact via pairwise van der Waals interactions $V(r) = -Cr^{-6}$, where C is a constant.

(a) What is the interaction energy between the plates as a function of h?

(b) How does this simplify in the limits of $d/h \ll 1$ and $d/h \gg 1$?

(c) How thick do the plates have to be so that the interaction energy is practically (within 1%) of that between two semi-infinite blocks?

Problem 3: Van der Waals Forces on an AFM Tip. Consider an AFM tip of radius R attached to a cantilever spring. The AFM tip interacts with the surface according to *a vdW interaction between a sphere and a flat surface*, which should be assumed to be attractive as discussed in the class. The height of the undeflected tip above the surface is H and the height of the deflected tip is h. Such deflection offers a resistance to the attraction like that of a Hookean spring, given by $k(H - h)$, where k is the spring constant.

(a) Pull-in instability: At what critical distance $H_{\min}$ does the tip touch the surface?

(b) After the tip touches and makes contact with the surface with a fixed gap g, what H is required to pull the tip off the surface?

(c) Describe how you would measure the Hamaker constant if you can easily measure the deflection.