Problem 1: R curve. A fairly tough steel has a plane strain fracture toughness of $K_{\rm Ic} = 100\ {\rm MPa}\sqrt{\rm m}$ and a resistance curve given by

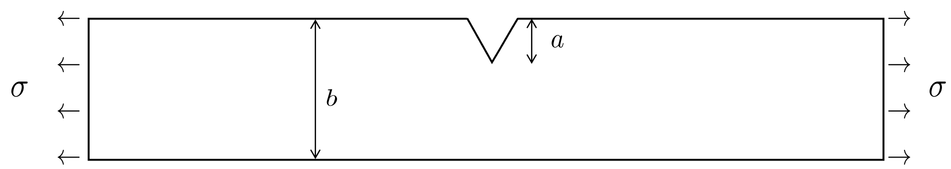
$$K_R(\Delta a) = K_{\rm Ic}\left[4 - 3\exp\left(-\Delta a/\ell\right)\right],$$

where $\ell = 3\ {\rm mm}$. An edge crack exists in the wall of a thick pressure vessel (in fact it is a nuclear pressure vessel). Determine the stress σ_c at which the crack starts to advance and the stress σ_* and the associated crack length increase Δa_* at which crack growth becomes unstable (assuming that the applied stress σ is specified). Take the thickness of the wall to be $b = 5\ {\rm cm}$ and the initial crack length to be $a = 1\ {\rm cm}$. From the Tada et al. Handbook, the stress intensity factor for this geometry and loading is

$$K_{\rm I} = \sigma\sqrt{\pi a}\, F(a/b),$$

where

$$F(a/b) = 0.265(1 - a/b)^4 + (0.857 + 0.265a/b)(1 - a/b)^{-3/2}.$$



Carry out this analysis assuming that small scale yielding (SSY) applies. How large would the tensile yield strength of the steel have to be for the assumptions of SSY to be valid?

Problem 2: Fatigue. A cylindrical steel pressure vessel of 7.5 m diameter and 40 mm wall thickness is to operate at a working pressure of 5.1 MPa. The design assumes that failure will occur by fast fracture from a crack that has extended gradually through the thickness of the vessel wall by fatigue. To prevent fast fracture, the total number of loading cycles from zero to full load and back to zero again must not exceed 3000. The fracture toughness for the steel is 200 ${\rm MPa}\sqrt{\rm m}$. The fatigue crack growth rate can be approximated using the Paris law such that

$$\frac{{\rm d}a}{{\rm d}N} = A\left(\Delta K_{\rm I}\right)^4,$$

where $A = 2.44 \times 10^{-14}\ ({\rm MPa})^{-4}{\rm m}^{-1}$. Find the minimum pressure to which the vessel must be tested before use to guarantee against failure in under the 3000 load cycles.

Problem 3: The relationship between K and J. Use the sum of the singular asymptotic fields for isotropic linear elastic solids under combined Mode I, II, and III loading. Use a circular integration path centered on the crack tip to derive the expression:

$$J = \frac{K_{\rm I}^2}{E'} + \frac{K_{\rm II}^2}{E'} + \frac{K_{\rm III}^2}{2\mu}$$

Include your Mathematica code if you have used it.

Problem 4: Energy release rate. The linear elastic specimen pictured below is loaded by two forces P on the left. The specimen has a thickness t. Treat this as a problem in plane stress and use the J-integral to evaluate the energy release rate $\mathcal{G}$. Also show that $K_{\rm II} = P/(2t\sqrt{b})$. Assume that $\ell \gg b$.