**Problem 1:** The Westergaard stress function for a center crack located at $(|x| < a, y = 0)$ loaded by equal and opposite point loads of magnitude $P$ located at $(x = b, y = \pm 0)$ has the form

$$Z_{\mathrm{I}}(z) = \frac{A + iB}{\sqrt{z^2 - a^2}(z - b)},$$

where $A$ and $B$ are real coefficients. Apply the following procedure to determine $A$ and $B$.

First, determine the $\sigma_{xx}$, $\sigma_{yy}$, and $\sigma_{xy}$ stress components from $Z_{\mathrm{I}}$. Apply the appropriate crack face boundary conditions to determine one of the constants. Consider a semi-circular arc around the point force on the top crack face. Set $z = b + re^{i\theta}$. To make the calculation as simple as possible, we need only to consider the limit as $r \to 0$ (but conceptually $r$ remains finite). In this limit, $z \to b$, $z - a \to b - a$, $z + a \to b + a$, and $z - b \to re^{i\theta}$. Finally, to determine the remaining constant, use the fact that the net force in the y-direction on this arc must be equal to $-P$, i.e.,

$$F_y = \int_0^\pi (\sigma_{xy} \cos\theta + \sigma_{yy} \sin\theta)\, r \,\mathrm{d}\theta = -P.$$

What are the values of $K_{\mathrm{I}}$ on the right and left crack tips? Does this agree with our results from class?

**Problem 2:** Apply the results of the Mode I weight functions for a semi-infinite crack to determine $K_{\mathrm{I}}$ for a semi-infinite crack loaded by equal and opposite point loads of magnitude $P$ located on the crack faces at a distance $b$ behind the crack tip. Use the results from Problem 1, and make an intelligent guess at what the Westergaard stress function is for this problem. Do your $K_{\mathrm{I}}$ values from the weight function and Westergaard stress function methods agree?

**Problem 3:** Show that the Mode II weight functions for a semi-infinite crack.