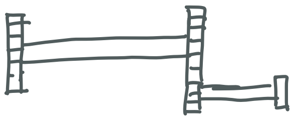
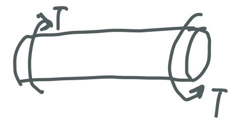
§3.1 圆截面直杆的扭转



直杆端部所受载荷可以简化为作用在轴向的力偶（扭矩）

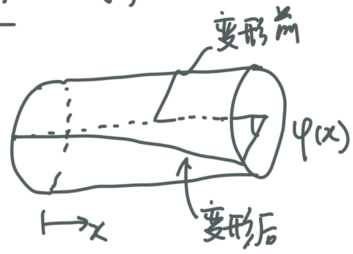
或



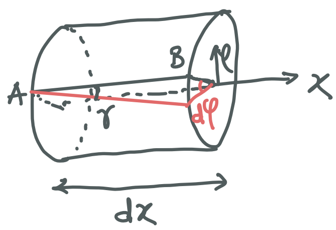
截面应力的合力（内力）为扭矩 T，该受力状态为扭转.

• <u>平截面假设</u>：横截面像刚性平面一样绕杆的轴线转动（几何大为简化但合理）

• <u>扭转角</u>（几何）



两个截面相对转动的角度 $\varphi=\varphi(x)$，取一微元 dx，得 $d\varphi$.

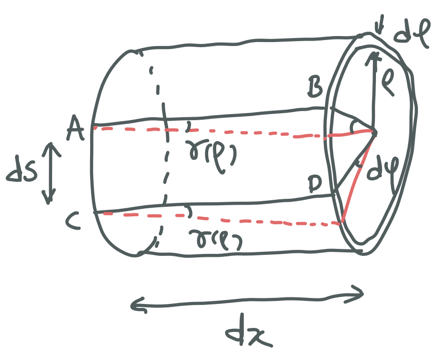


依赖载荷

单位长度的扭转角为 $\theta = \frac{d\varphi}{dx} \rightarrow d\varphi = \theta(x)\,dx$

AB 的长度变化 $O(\gamma^2)$，角度变化 $\gamma = \gamma(\rho)$

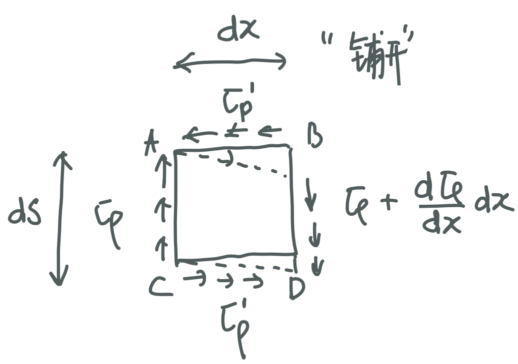
径向位置



在 ρ 处，取一 $d\rho$ 厚度圆筒

$$\gamma(\rho) = \frac{d\varphi \times \rho}{dx} = \theta\cdot\rho \rightarrow \boxed{\gamma(x,\rho) = \theta(x)\rho}$$

ABDC 的切应变（无正应变）



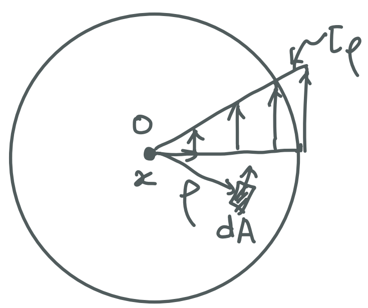
取出微元 $dx\ ds$（纯剪切状态）

$\tau_\rho' = \tau_\rho$（力矩平衡）

$\boxed{\tau_\rho = G\gamma = G\theta\rho}$（物理方程/胡克材料）

考虑完几何和物理方程后，还有平衡方程!!! 首先考查内力.

横截面



$\sigma_x \equiv 0$

$$N_x=0=Q_y=Q_z=M_y=M_z$$

$$M_x=\int_A \tau_\rho dA\cdot\rho = G\theta\underbrace{\int_A \rho^2 dA}_{\text{极惯性矩 } I_p}$$

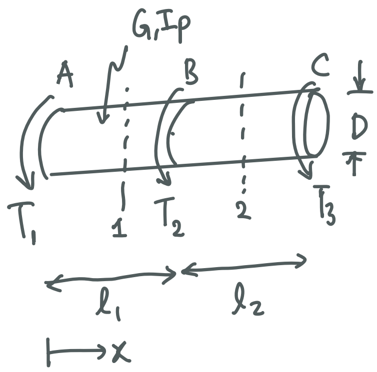
对于圆截面 $I_p = 2\pi\int_0^{\frac{D}{2}}\rho^2\rho\, d\rho = \frac{\pi}{32}D^4$

单位长度扭角-扭矩关系: $\theta(x)=\dfrac{M_x(x)}{GI_p}$

扭转切应力: $\tau_\rho(x,\rho)=\dfrac{M_x\rho}{I_p}$ （0到 $\dfrac{M_xD}{2I_p}$ 线性变化）

当M_x=常数时, 扭角-扭矩关系: $\varphi=\dfrac{M_x l}{GI_p}$, GI_p — 扭转轴截面刚度, $\dfrac{GI_p}{l}$ — 扭转刚度

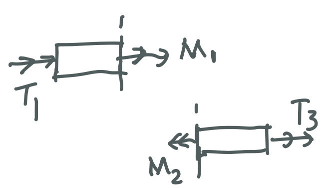
例 1.



$T_1=-1592\ N\cdot m$, $T_2=955\ N\cdot m$, $T_3=637\ N\cdot m$

求最大切应力，相对扭转角.

① 平衡方程



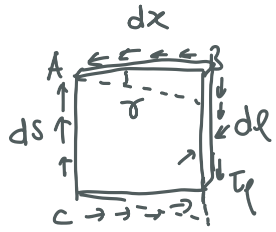
$M_1=-T_1$, $M_2=T_3$ or $M_x=\begin{cases}-T_1, & 0\le x<l_1\\ T_3, & l_1<x\le l_1+l_2\end{cases}$

$$\tau_{max}=\frac{|M_x|_{max}\rho_{max}}{I_p}=\frac{|T_1|D}{2I_p}$$

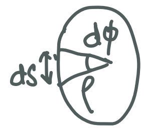
② 物理方程

$$\varphi_1=\frac{M_1l_1}{GI_p},\quad \varphi_2=\frac{M_2l_2}{GI_p}\ \rightarrow\ \varphi_{AC}=\varphi_1+\varphi_2=\frac{-T_1l_1+T_3l_2}{GI_p}$$

• 扭转应变能



$dV = ds\, d\rho\, dx$



微元的应变能为 $\underbrace{\frac{1}{2} \times \tau_\rho d\rho \underset{\uparrow\ \rho d\phi}{ds} \times \underset{\uparrow\ \frac{1}{G}\tau_\rho}{\gamma} dx}_{\frac{1}{2}\tau_\rho\gamma dV} = \frac{1}{2G}\tau_\rho^2 \rho\, d\phi\, d\rho\, dx = u_1$

(dx长)圆筒的应变能为 $\int_0^{2\pi} u_1 d\phi = \frac{\pi}{G}\tau_\rho^2 \rho\, d\rho\, dx = \frac{\pi}{G}\frac{M_x^2\rho^3}{I_p^2} d\rho\, dx = u_2$

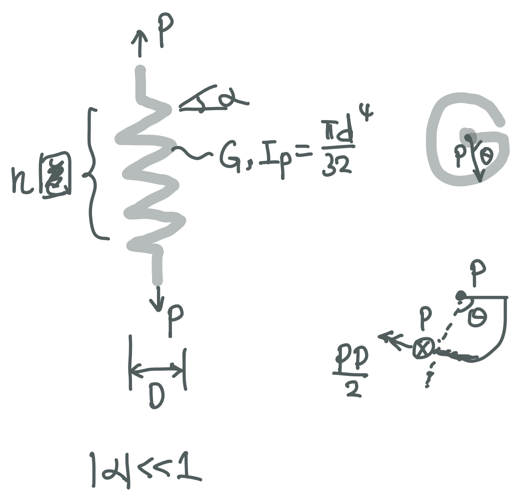
(dx长)圆轴的应变能为 $\int_0^{\frac{D}{2}} u_2 d\rho = \frac{\pi}{G}\frac{M_x}{4I_p^2}\left(\frac{D}{2}\right)^4 \cdot dx = \frac{1}{2G}\frac{M_x^2}{I_p}dx$ $\left(I_p = \frac{\pi}{32}D^4\right)$

整个轴的应变能为 $U = \int_V \frac{1}{2}\tau_\rho\gamma\, dV$ (单位体积)

$= \int_0^l \frac{1}{2}\frac{M_x^2}{GI_p}dx = \int_0^l \frac{1}{2}M_x \cdot \theta\, dx = \int_0^l \frac{1}{2}GI_p\theta^2 dx$ (单位长度)

$= \frac{M_x^2 l}{2GI_p} = \underbrace{\frac{1}{2}M_x\theta l}_{W=\frac{1}{2}M_x\varphi\ \text{外力功}} = \frac{1}{2}GI_p l\theta^2$ (M_x, θ 为常数时)

例2. 密圈螺旋弹簧



忽略P剪力对应的变形能.

$W = \frac{1}{2}P\Delta$

$U = n \times \frac{1}{2}\frac{M_x^2 l}{GI_p} = n \times \frac{1}{2} \times \frac{P^2D^2}{4} \times \frac{\pi D}{G\frac{\pi d^4}{32}} = \frac{4nP^2D^3}{Gd^4}$

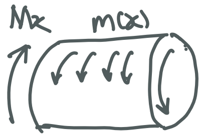
$W = U \rightarrow \Delta = \underbrace{\frac{8nD^3}{Gd^4}}P$

$\rightarrow k = \frac{Gd^4}{8nD^3}$ 弹簧刚度

(需要 $D \gg d$ 才可忽略剪力P)

· 非均匀扭转

现在考虑更为一般的形式 $M_x=M_x(x)$，$\theta=\theta(x)$，$I_p=I_p(x)$



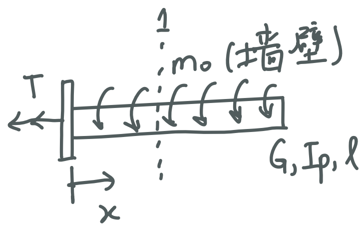
$$M_x+\frac{dM_x}{dx}dx+O(dx^2) \rightarrow \boxed{\frac{dM_x}{dx}+m(x)=0}$$ 平衡方程

物理方程 仍为 $\theta=\frac{M_x}{GI_p}=\frac{d\varphi}{dx}$ (因为正是根据微元FBD分析得到的).

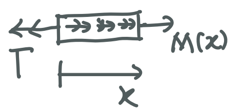
$$\rightarrow \boxed{\varphi(l)-\varphi(0)=\int_0^l \frac{M_x(x)}{G(x)I_p(x)}dx}$$

同样地：$\tau_\rho(x,\rho)=\frac{M_x(x)\rho}{I_p(x)}$

例3. 螺钉受摩擦作用



① 平衡方程



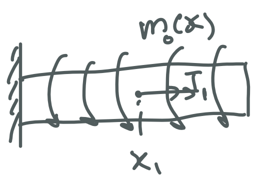
$T=m_0 l$

$M(x)=T-m_0x=m_0(l-x)$

② 物理方程

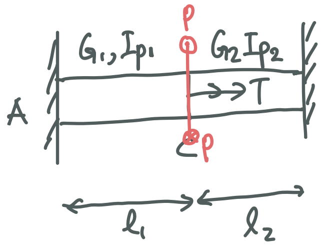
$$\theta=\frac{d\varphi}{dx}=\frac{M}{GI_p}=\frac{m_0(l-x)}{GI_p}$$

$$\Delta\varphi=\int_0^l\theta\,dx=\frac{m_0l^2}{2GI_p}$$

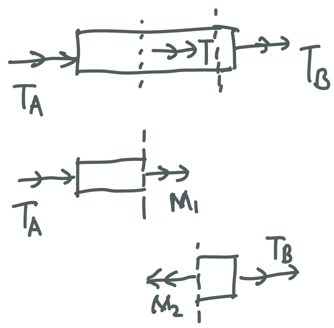


? 分段或 $m(x)=m_0(x)+T_1\delta(x-x_1)$

· 扭转静不定问题



① 平衡方程



$$T_A + T_B + T = 0$$

$$M_x = \begin{cases} -T_A, & 0 \le x < l_1 \\ T_B, & l_1 < x \le l_1 + l_2 \end{cases}$$

② 物理方程

$$\varphi_{AC} = \frac{-T_A l_1}{G_1 I_{p1}}, \quad \varphi_{CB} = \frac{T_B l_2}{G_2 I_{p2}}$$

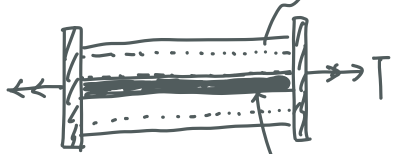
Check:
$k_1 \to \infty$, k_2 finite?

③ 几何方程

$$\varphi_{AB} = \varphi_{AC} + \varphi_{CB} = 0 \to T_A = \frac{-\frac{G_1 I_{p1}}{l_1}}{\underbrace{\frac{G_1 I_{p1}}{l_1}}_{k_1} + \underbrace{\frac{G_2 I_{p2}}{l_2}}_{k_2}} T = -\frac{k_1 T}{k_1 + k_2}, \quad T_B = -\frac{k_2 T}{k_1 + k_2}, \quad \varphi_{AC} = -\varphi_{CB} = \frac{T}{k_1 + k_2}$$

套筒组合轴

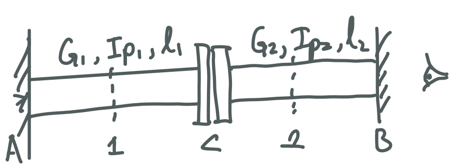
筒 $G_1, I_{p1} = \frac{\pi}{32}(D_{out}^4 - D_{in}^4)$

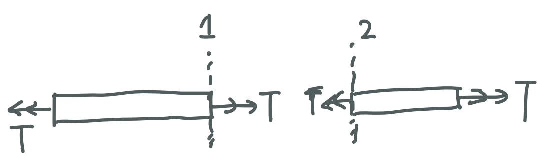


杆 G_2, I_{p2}

$$\underbrace{\Delta\varphi_{筒} = \Delta\varphi_{杆}}_{几何方程} = \frac{Tl}{G_1 I_{p1} + G_2 I_{p2}}$$

例4. 装配扭矩

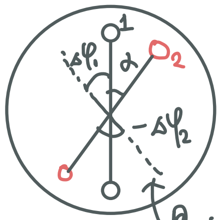




物理: $\Delta\varphi_1 = \frac{T l_1}{G_1 I_{p1}}$, $\Delta\varphi_2 = \frac{T l_2}{G_2 I_{p2}}$

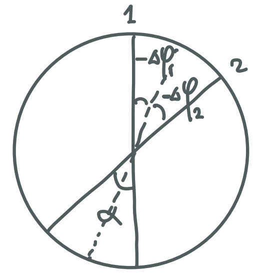
$\phi_c \quad \phi_c'$

$\phi_c - \phi_A = -\alpha$



最终状态

几何: $-\Delta\varphi_2 = \Delta\varphi_1 + \alpha$



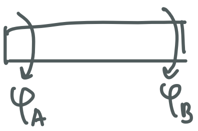
$$\to T = \frac{-\alpha}{l_1/G_1I_{p1} + l_2/G_2I_{p2}}$$

$$\Delta\varphi_1 = \frac{-l_1/G_1I_{p1}\,\alpha}{l_1/G_1I_{p1} + l_2/G_2I_{p2}}, \quad \Delta\varphi_2 = \frac{-l_2/G_2I_{p2}\,\alpha}{l_1/G_1I_{p1} + l_2/G_2I_{p2}}$$

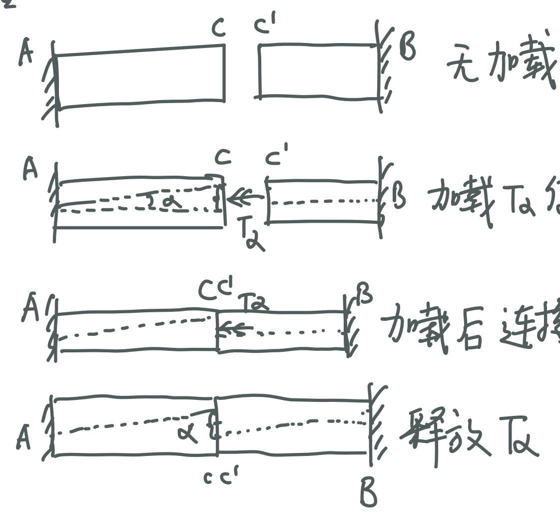
Check: ① $l_1/G_1I_{p1} = l_2/G_2I_{p2} \longrightarrow \Delta\varphi_1 = \Delta\varphi_2 = -\frac{\alpha}{2}$ ✓

② $l_1/G_1I_{p1} \to 0 \longrightarrow \Delta\varphi_1 \to 0,\ \Delta\varphi_2 \to -\alpha,\ T \to \frac{-\alpha G_2I_{p2}}{l_2}$ ✓

更为简单一些的几何考量



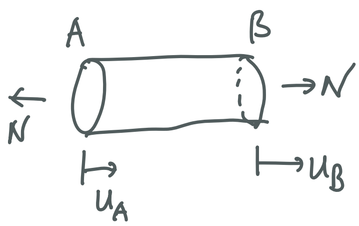
$\Delta\varphi = \varphi_B - \varphi_A$

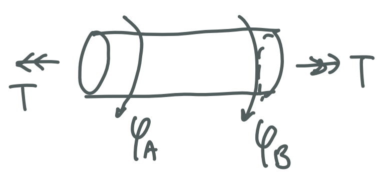


$$\boxed{\varphi_C' - \varphi_C = \alpha}$$

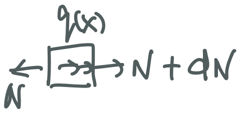
$$\underbrace{\varphi_C - \cancel{\varphi_A}^{0}}_{\Delta\varphi_1} + \underbrace{\cancel{\varphi_B}^{0} - \varphi_C'}_{\Delta\varphi_2} + \alpha = 0$$

拉压问题与扭转问题的相似性





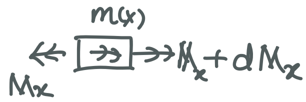
- $\underbrace{\Delta l = u_B - u_A = \frac{Nl}{EA}}_{\text{均 } N,\ E,\ A}$
- $\underbrace{\Delta\varphi = \varphi_B - \varphi_A = \frac{Tl}{GI_p}}_{\text{均 } T,\ G,\ I_p}$



$$\frac{dN}{dx}+q(x)=0$$

$$\varepsilon(x)=\frac{du}{dx}=\frac{N(x)}{EA}$$

$$\underbrace{\Delta l(x)=\int_0^x \frac{N(x)}{E(x)A(x)}\,dx}_{\text{非均匀 } N,E,A}$$

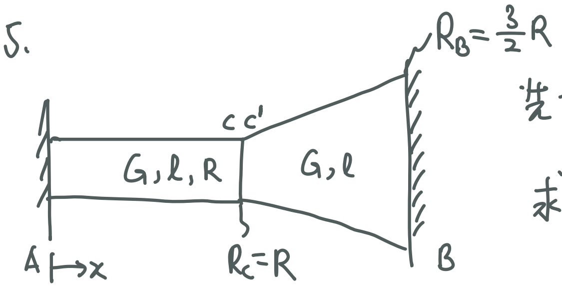


$$\frac{dM_x}{dx}+m(x)=0$$

$$\theta(x)=\frac{d\varphi}{dx}=\frac{M_x(x)}{GI_p}$$

$$\underbrace{\Delta\varphi(x)=\int_0^x \frac{M_x(x)}{G(x)I_p(x)}\,dx}_{\text{非均匀 } M_x,\ G,\ I_p.}$$

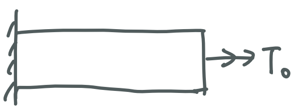
例5.



先施加 ϕ_0 到 AC，再固定 cc'

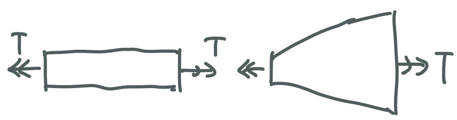
求 $\varphi(x)$ 分布.

①



$$\varphi_0=\varphi_c-\cancel{\varphi_A}^{0}=\frac{T_0 l}{GI_p}\rightarrow T_0=GI_p\frac{\phi_0}{l}$$

②



杆AC: $\phi_c-\cancel{\phi_A}^{0}=\frac{Tl}{GI_p}\rightarrow\phi_c=\frac{2Tl}{\pi GR^4}\rightarrow\phi(x)=\frac{2Tx}{\pi GR^4}$

杆CB (具有非均匀 I_p): $R(x)=R+\frac{1}{2}R\frac{x-l}{l}\rightarrow I_p(x)=\frac{\pi}{2}R^4(x)$

$$\cancel{\phi_B}^{0}-\phi(x)=\int_x^{2l}\frac{2T}{G\pi\left[R+\frac{1}{2}R\frac{x-l}{l}\right]^4}\,dx$$

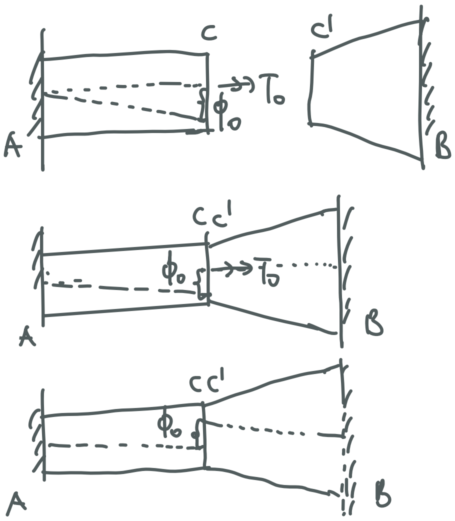
$u=R+\frac{1}{2}R\frac{x-l}{l}$

$du=\frac{1}{2}\frac{R}{l}dx$

$$=\int_{R+\frac{1}{2}\frac{R}{l}(x-l)}^{\frac{3}{2}R}\frac{2T}{\pi Gu^4}\cdot\frac{2l}{R}\,du$$

$$=\frac{4Tl}{3\pi GR}\left[-u^{-3}\right]_{R+\frac{1}{2}\frac{R}{l}(x-l)}^{\frac{3}{2}R}=-\frac{4Tl}{3\pi GR}\left[\frac{1}{\left(R+\frac{1}{2}R\frac{x-l}{l}\right)^3}-\frac{8}{27}\frac{1}{R^3}\right]$$

$$\rightarrow \phi_{c'} = \phi(x=l) = -\frac{4Tl}{3\pi GR}\cdot\frac{19}{27}\frac{1}{R^3} = -\frac{76Tl}{81\pi GR^4}$$



几何方程 $\phi_c - \phi_{c'} \equiv \phi_0$

$$\rightarrow \frac{2Tl}{\pi GR^4} + \frac{76Tl}{81\pi GR^4} = \phi_0 \rightarrow T = \frac{81}{238}\frac{\pi GR^4}{Tl}\phi_0$$

Check: ① 期待 $\phi_{c'}<0$，$T>0 \rightarrow \phi_{c'}<0$ ✓

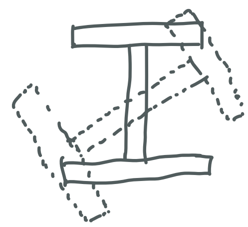
② $0<\phi_c<\phi_0$，$\phi_c = \frac{81}{119}\phi_0$ ✓

$$\tau_{max} = \left(\frac{TR}{GI_p}\right)_{max} = \frac{2T}{\pi G R^3}\Big|_{max} \rightarrow \text{在AC杆表面处}$$

§3.2 闭口薄壁截面直杆的扭转

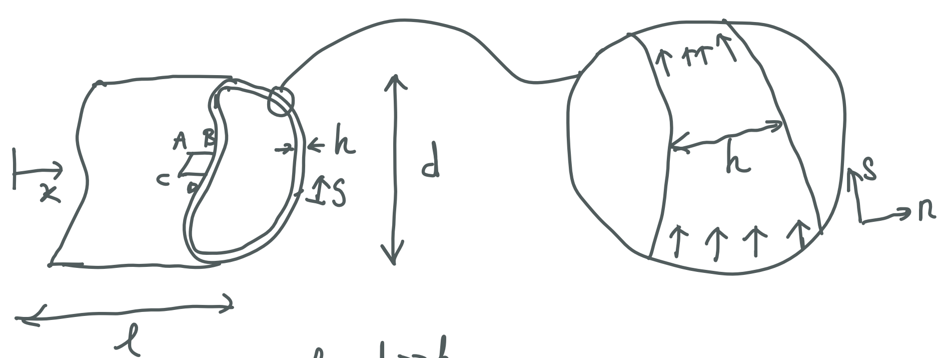
· 刚周边假设

对于非圆截面，平截面假设不再合适。例如，同样面积下，圆的 I_p 小于矩形截面的 I_p 但实际圆截面杆的抗扭刚度更大？截面发生翘曲。接下来，我们允许截面翘曲，但假设翘曲后的截面在其变形前的平面上的投影形状保持不变。



扭转变形可分成两部分 ①绕杆轴转过一个角度 ②沿杆轴方向翘曲.

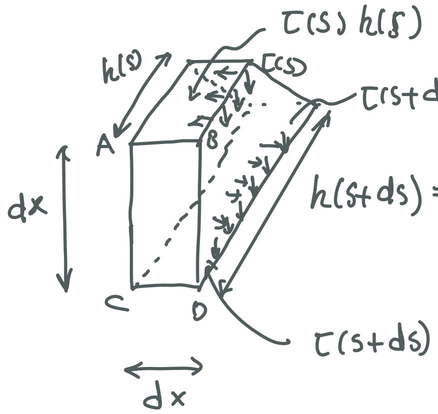
· 薄壁截面



$l \gg d \gg h$

认为应力 τ 沿厚度 h 方向的变化不重要，更关心 τ 作为厚度方向的平均，对应的单位 s 的合力 τh.

先考虑均匀扭转(不依赖于x) → $\tau=\tau(s)$, $h=h(s)$，取微元ABCD.



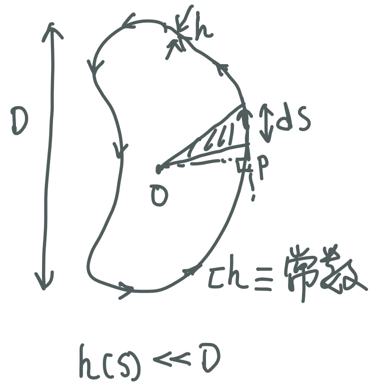
$$\tau(s+ds) = \tau + \frac{d\tau}{ds}ds$$

$$h(s+ds) = h + \frac{dh}{ds}ds$$

$$\Sigma F_x = 0 \rightarrow \boxed{\frac{d(\tau h)}{ds} = 0}$$

$$\tau(s+ds)\,h(s+ds) = \tau h + \underbrace{\frac{\partial \tau}{\partial s} h\,ds + \tau\frac{\partial h}{\partial s}ds}_{\frac{d(\tau h)}{ds}ds} + O(d^2 s)$$

可以进一步求τh所带来的合力矩(内力)

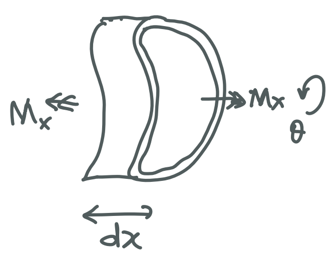


$dM_x = op\cdot \tau h\,ds = 2(\tau h)\,dA_m$ ← 阴影面积.

$$\rightarrow M_x = 2\tau h \underbrace{A_m}_{\text{截面中线所包围的面积}} \quad \text{or} \quad \boxed{\tau(s) = \frac{M_x}{2A_m h(s)}}$$

截面中线所包围的面积　τ_{max} 发生于 h_{min} 处，τ_{min} 发生于 h_{max} 处

怎么确定τ-θ 或 M-θ关系? 通常采用微元分析(确实可以P107-108)，但我们在这里采用一个更简洁的方法- 能量法.



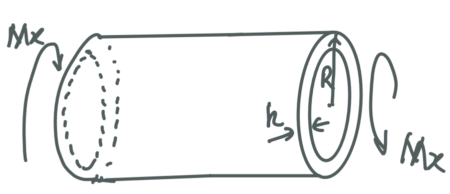
$$\underbrace{\frac{1}{2}M_x\theta\,dx}_{\text{整体层面或外力功}} = \frac{1}{2}\int_V \frac{\tau^2}{G}dV = \frac{1}{2}\oint\frac{\tau^2}{G}dx\cdot ds\cdot h$$

$$= \frac{\tau h}{2G}\oint \tau\,ds$$

$$= \frac{M_x}{4GA_m}\oint \tau\,ds \Rightarrow \boxed{\theta = \frac{1}{2GA_m}\oint \tau\,ds}$$

$$\theta = \frac{M_x}{G\left(4A_m^2/\oint\frac{1}{h}ds\right)}$$

例1. 薄壁圆筒



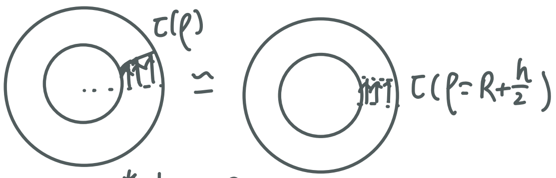
$A_m = \pi R^2$, $h(s)\equiv h$

$$\rightarrow \tau(s) = \frac{M_x}{2\pi R^2 h}, \quad \theta = \frac{1}{2G\pi R^2}\overbrace{\oint \tau\,ds}^{\frac{M_x}{2\pi R^2}\cdot\frac{1}{h}\cdot 2\pi R} = \frac{M_x}{G\cdot\underbrace{2\pi R^3 h}_{I_p?}}$$

圆截面可以用 $\theta=\frac{M_x}{GI_p}$ 可求解，其中 $I_p=\frac{\pi}{32}(D_{out}^4-D_{in}^4)$

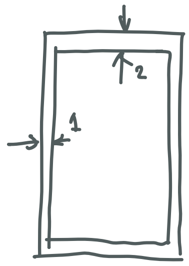
$$=\frac{\pi}{2}(R_{out}^4-R_{in}^4)$$

$$=\frac{\pi}{2}\underbrace{\left[(R+\tfrac{h}{2})^4-(R-\tfrac{h}{2})^4\right]}_{(R^4)'\cdot h}\simeq 2\pi R^3 h$$



当 $h\ll R$ 时

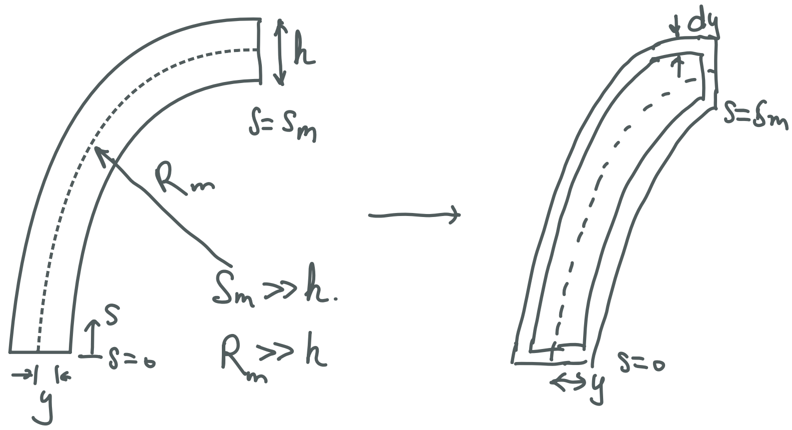
例2.



τ_{max} ? (P110).

§3.3. 开口薄壁截面直杆的扭转

该类问题的解法可参考弹性力学。这里我们可以做一定简化，方便工程设计。



· 取距离截面中心线 y 处的 dy 厚度"闭口薄壁杆"。($\tau dy=$ 常数).

· 认为任意 y 处的"闭口杆"都扭转 θ (刚截面)

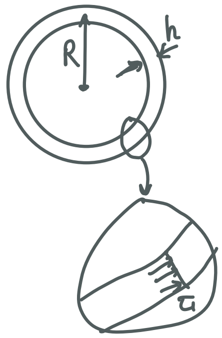
$$\theta=\frac{1}{2GA_m}\oint\tau ds\ ,\quad A_m(y)\simeq 2S_m y\ ,\quad \oint\tau ds\simeq 2S_m\tau$$

$$\to\ \theta\simeq\frac{2S_m\tau}{2G\cdot 2S_m y}=\frac{\tau(y)}{2Gy}\quad \text{or}\quad \boxed{\tau(y)=2G\theta y}\ \text{线性分布.}$$

$$\tau = \frac{dM_x}{2A_m dy} \rightarrow dM_x = 2\cdot 2S_m y \cdot 2G\theta y \cdot dy = 8GS_m\theta y^2 dy$$

$$\rightarrow \boxed{M_x = \frac{1}{3}GS_m h^3\theta \quad \text{or} \quad \theta = \frac{M_x}{G\frac{1}{3}S_m h^3}}$$

无缝管

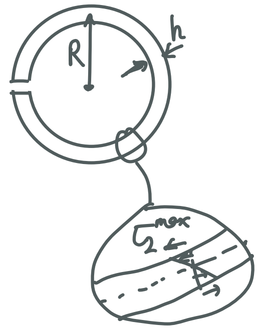


$$\theta_1 = \frac{M_x}{G 2\pi R^3 h}$$

$$\tau_1 = \frac{M_x}{2\pi R^2 h}$$

$$k_1 = \frac{M_x}{\theta_1 l} = 2\pi R^3 h/l$$

有缝管



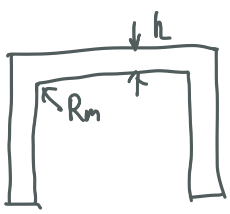
$$\theta_2 = \frac{M_x}{G\frac{1}{3}2\pi R\cdot h^3}$$

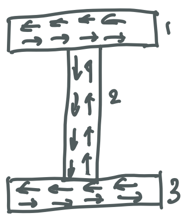
$$\tau_2^{max} = \frac{M_x}{\frac{1}{3}\pi R h^3}\times\frac{h}{2}$$

$$k_2 = \frac{M_x}{\theta_2 l} = \frac{2}{3}\pi R h^3/l.$$

$$\begin{cases} k_1/k_2 = 3R^2/h^2 >>> 1. & I_{p1} \sim R^3 h \\ \tau_1/\tau_2^{max} = h/3R << 1 & I_{p2} \sim R h^3 \end{cases}$$

工程中截面的形状往往是几个开口杆的拼接，如工字梁．拼接处的曲率半径可以认为是无限小，这时的处理方式为假设每一部分承受的扭转是独立的（但θ相同）.





$$(M_x)_i = \frac{\theta}{3}(GS_m h^3)_i$$

$$M_x = \sum_i^n (M_x)_i = \frac{\theta}{3}\sum_i (GS_m h^3)_i$$

编号k部分上扭矩：$(M_x)_k = \dfrac{(GS_m h^3)_k M_x}{\sum_i (GS_m h^3)_i}$

编号k部分上最大切应力 $(\tau_k)_{max} = \dfrac{3(Gh)_k M_x}{\sum_i (GS_m h^3)_i}$

对于 $G_1 = G_2 = \cdots G_n$，$\tau_{max} = \dfrac{3M_x h_{max}}{\eta\sum_i (S_m h^3)_i}$

η：修正系数 ~1.2（工字钢）

§3.4. 直杆扭转的强度和刚度计算.

强度：$\tau = \dfrac{M_x \rho}{I_p} \rightarrow |\tau|_{max} = \dfrac{|M_x|_{max}}{W_p} \leq [\tau] \sim \begin{cases} 0.5 \sim 0.6\ [\sigma] & \text{塑性材料} \\ 0.8 \sim 1.0\ [\sigma] & \text{脆性材料.} \end{cases}$

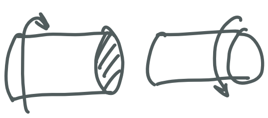
W_p：抗扭截面系数

刚度：$\theta = \dfrac{M_x}{G I_p} \rightarrow |\theta|_{max} = \dfrac{|M_x|_{max}}{G C} \leq [\theta] \sim \begin{cases} 0.15^\circ \sim 0.30^\circ/m & \text{精密机器} \\ 2^\circ/m & \text{一般传动轴.} \end{cases}$

C：几何刚度系数

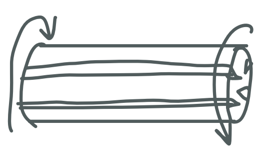
	W_p	C
圆截面	$\frac{1}{16}\pi D^3$	$\frac{1}{32}\pi D^4$
圆筒截面	$\frac{1}{16}\pi D^3\left(1-\frac{D'^4}{D^4}\right)$	$\frac{1}{32}\pi\left(D^4-D'^4\right)$
闭口薄壁	$2A_m h_{min}$	$4A_m^3 / \oint \frac{1}{h} ds$
开口薄壁	$\frac{\eta}{3 h_{max}} \sum_i (S_m h^3)_i$	$\frac{1}{3}\eta \sum_i (S_m h_i^3)_i$

• 破坏模式



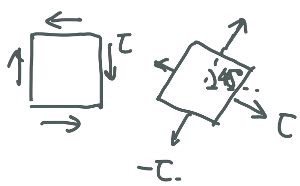
低碳钢

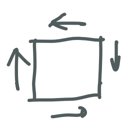
铸铁(骨头)

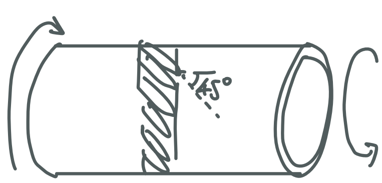


木材

$\tau \rightarrow \tau_s$



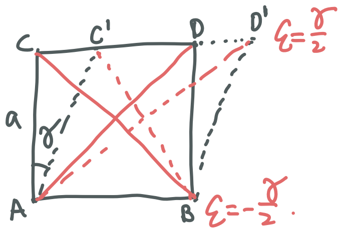




薄壁管(失稳)

这是一个很好的应力分析的例子. 我们已经讨论过 这个例子. 现在讨论下面这个例子:

作业题 1.7

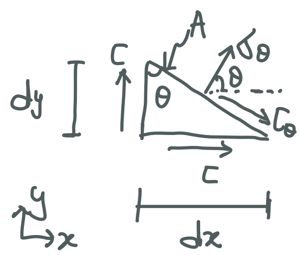


$$AD' = \sqrt{a^2+a^2(1+\gamma)^2} = \sqrt{2}a\sqrt{1+\gamma+\frac{1}{2}\gamma^2} \simeq \sqrt{2}a\left[1+\frac{1}{2}\gamma+O(\gamma^2)\right]$$

$$\varepsilon_{AD} = \frac{AD'-AD}{AD} = \frac{1}{2}\gamma$$

$$\varepsilon_{BC} = -\frac{\gamma}{2}$$ (相同的推导方法)

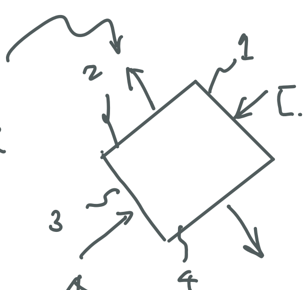
纯剪切的45°方向会有线应变 $\pm\frac{1}{2}\gamma$.



$$\Sigma F_{\sigma_\theta}=0 \rightarrow \sigma_\theta A + \underbrace{\tau \cdot A\cos\theta}_{F_\uparrow}\cdot\sin\theta + \underbrace{\tau A\sin\theta}_{F_\rightarrow}\cdot\cos\theta = 0 \rightarrow \sigma_\theta = -\tau\sin 2\theta$$

$$\Sigma F_{\tau_\theta}=0 \rightarrow \tau_\theta A - \tau A\cos\theta\cdot\cos\theta + \tau A\sin\theta\,\sin\theta = 0 \rightarrow \tau_\theta = \tau\cos 2\theta$$

可能会带来拉伸破坏



可能会带来压缩失稳

$1: \theta=\frac{\pi}{4} \rightarrow \sigma_1=-\tau,\ \tau_1=0$

$2: \theta=\frac{3\pi}{4} \rightarrow \sigma_2=\tau,\ \tau_2=0$

$3: \theta=\frac{5}{4}\pi \rightarrow \sigma_3=-\tau,\ \tau_3=0$

$4: \theta=\frac{7}{4}\pi \rightarrow \sigma_4=\tau,\ \tau_4=0$