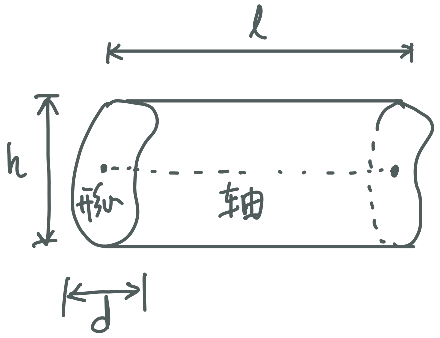
在本章，我们统一本课程所涉及的专业语言，并简要介绍相关的约定 (convention).

§1.1 研究对象、基本假设.

· **杆件**：由实际构件和结构简化而成的力学模型.



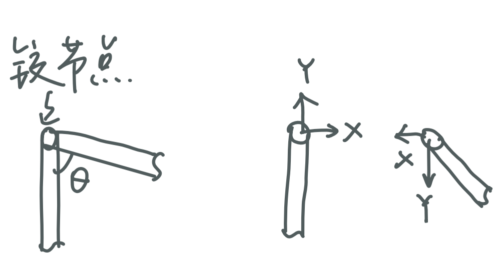
$h \sim d$，$h \ll l$ 杆

$h \gg d$，$h \ll l$ 薄壁杆

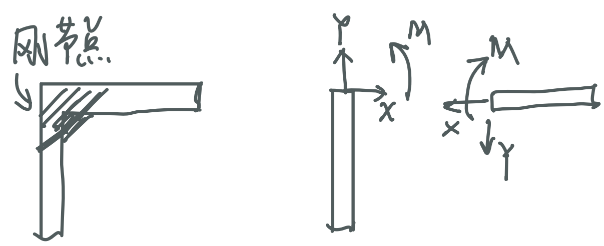
$h \gg d$，$h \sim l$ 板(壳)

系统总会存在一个小量 (i.e., $\varepsilon = h/l$)，使得我们可以简化三维理论/边界条件.

· **杆系**：由若干杆件通过节点连接组成的系统



夹角可自由改变 → 不能传递力矩



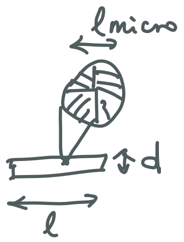
夹角保持不变 → 可传递力矩

· **基本假设**：

连续介质假设：物体在变形后不产生新的裂缝和空洞 (Not like Liberty ships).

均匀性假设：任意物质点具有连续、稳定的统计平均值

→ 力学参量是连续变化的，可用空间坐标的连续函数来表示

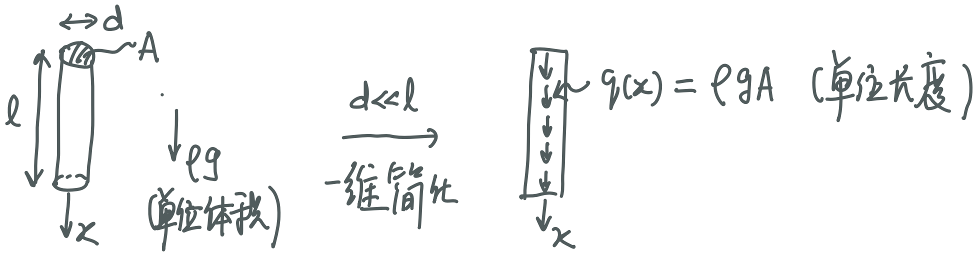


$$\min\{h, d, l\} \gg l_{micro}$$

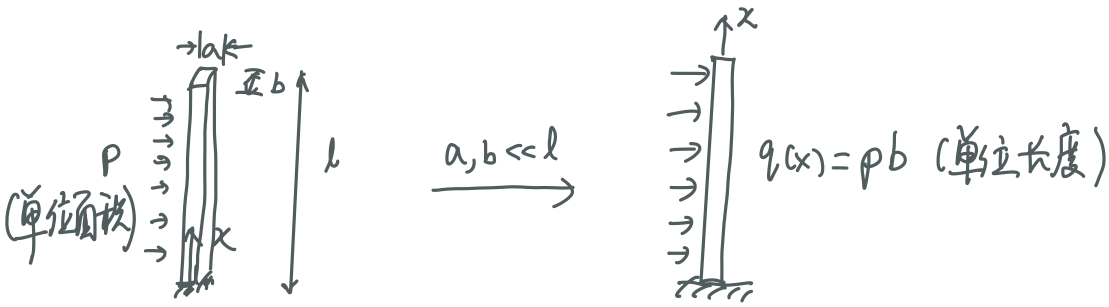
后续还会涉及到平截面假设，小变形假设，线弹性假设等..

载荷：(事先已知的)对所研究物体质点的作用力，也称主动力.

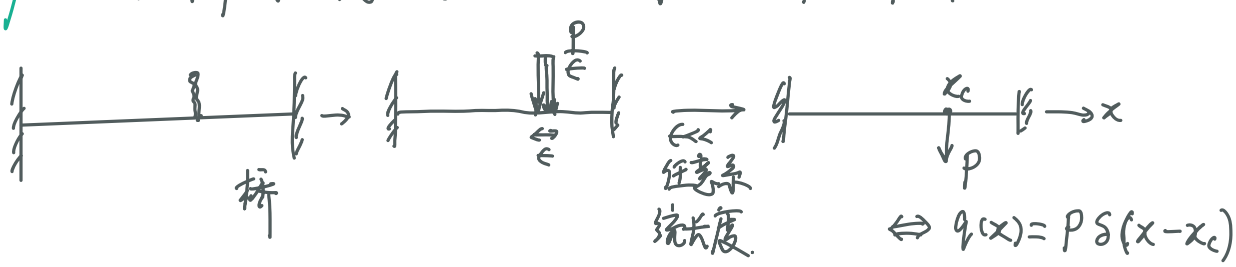
体积力：N/m^3，施加在物体的体积上. 如惯性力、磁力、重力



表面力：N/m^2，施加在物体的表面上，如水压力、风压力、摩擦力..



集中力：N，作用面积可以忽略的表面力，以合力的形式作用于一点.



(一定程度认为 x_c 点足够"安全"，更为关心在远离 x_c 处 P 所带来的影响).

约束：物体在空间上转/移动所受到的限制．对应的限制作用力为约束力（具体大小未知，依赖于具体的载荷）．(支反力)

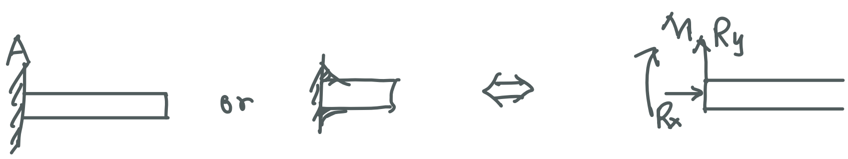
简支

A点保持不动　　A_x, A_y 使得 $\underbrace{X_A=Y_A}_{\text{位移}}=0$

活动简支

R_y 使得 $Y_A=0$

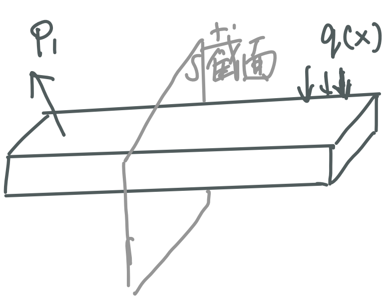
固支

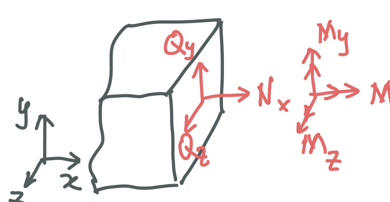


$\{M, R_x, R_y\}$ 使得 $X_A=Y_A=\theta_A=0$

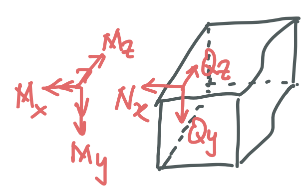
§1.2 内力

物体在外力作用下产生变形，但未发生断裂，这也与约束作用相同（也被称为内约束）．内约束力则为内力．可采用截面法显示内力——面两侧物质作用的合力(矩)．





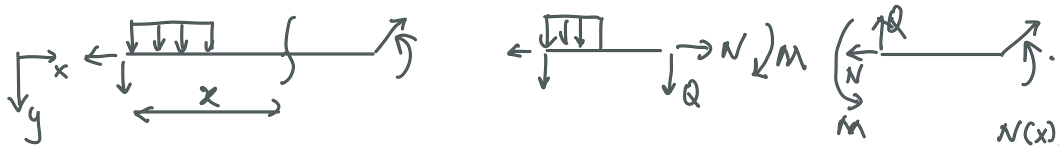
"左侧"约定方向



"右侧"约定方向

N_x - 轴力；Q_y, Q_z - 剪力；M_x - 扭矩；M_y, M_z - 弯矩.

2D:

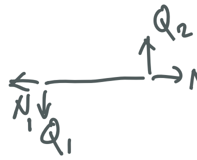


$N(x), Q(x), M(x)$?

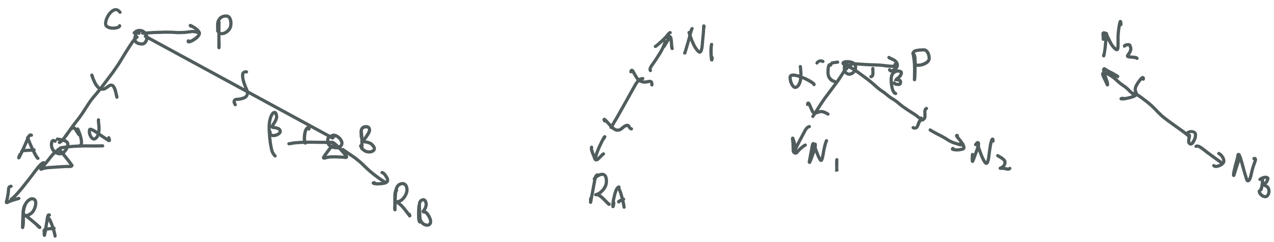
如何求内力和支反力？— 自由体（Free body diagram）

例1：两杆构成的简单桁架

桁架是由直杆铰接组成的杆系。无法传递力矩。



$\Rightarrow N_1 = N_2, Q_1 = Q_2 = 0$



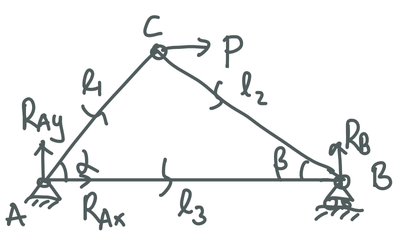
$$\rightarrow \begin{cases} N_1 = R_A, \quad N_2 = R_B \\ N_1 \cos\alpha = P + N_2 \cos\beta \\ N_1 \sin\alpha + N_2 \sin\beta = 0 \end{cases} \Rightarrow \quad N_1 = \frac{\sin\beta}{\sin(\alpha+\beta)} P, \quad N_2 = -\frac{\sin\alpha}{\sin(\alpha+\beta)} P$$

4方程，4未知数

线性 N-P 关系

实际方向与FBD标示而反（压缩）.

例2：三根杆构成的简单桁架



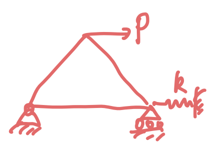
① 求支反力

$\Sigma F_x = 0 \rightarrow P + R_{Ax} = 0 \rightarrow R_{Ax} = -P$

$\Sigma F_y = 0 \rightarrow R_{Ay} + R_B = 0$

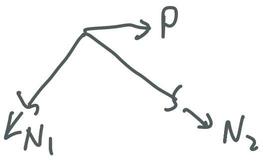
$\Sigma M_A = 0 \rightarrow R_B \cdot l_3 - P\, l_1 \sin\alpha = 0$

How about this?



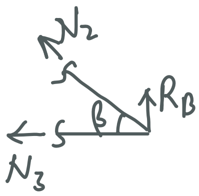
$\rightarrow R_B = \frac{l_1}{l_3} P \sin\alpha, \quad R_{Ay} = -\frac{l_1}{l_3} P \sin\alpha$

②求内力



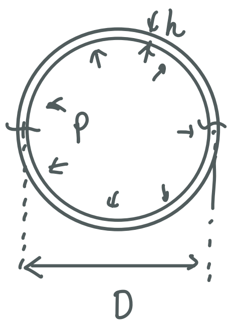
根据例1知: $N_1 = \frac{\sin\beta}{\sin(\alpha+\beta)} P$

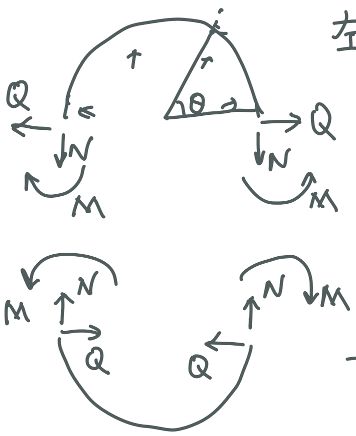
$$N_2 = -\frac{\sin\alpha}{\sin(\alpha+\beta)} P$$



$$N_3 = -N_2\cos\beta = \frac{\sin\alpha\cos\beta}{\sin(\alpha+\beta)} P.$$

例3. 受均布内压力p的薄壁圆环（"Boston糖罐"）





左右对称

上下对称 → $Q=0$

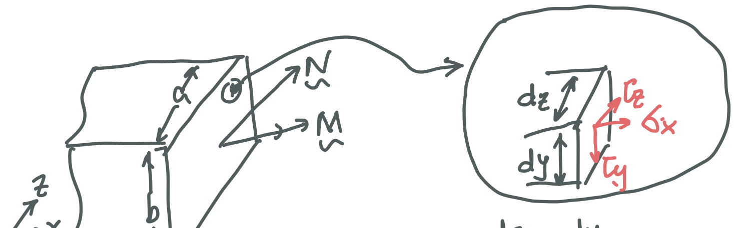
$$\Sigma F_y=0 \rightarrow 2N=\int_0^{\pi} q\sin\theta\,\frac{D}{2}\,d\theta = qD \rightarrow N=\frac{1}{2}qD$$

What is M? You will be able to show M not important as long as $h \ll D$.

§1.3 应力

内力实际上整个截面上的合力和合力矩，而截面上每个物质点的受力状态连续但通常依赖于具体的位置和外力情况。

应力.



$\frac{dz}{a}, \frac{dy}{b} \to 0$

σ_x — 正应力 (Normal stress)

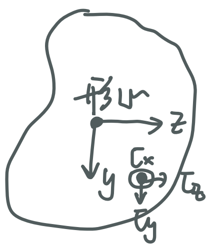
τ_y, τ_z — 切应力 (shear stress)

(更为常见的标示为 σ_{xx}, τ_{xy}, τ_{xz})

取自于 x 方向截面

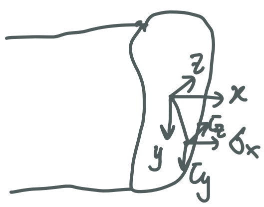
- 应力的量纲为 $[F]/[L^2]$，与压力相同.
- $\sigma_x = \sigma_x(y,z)\big|_x$, $\tau_y = \tau_y(y,z)\big|_x$, $\tau_z = \tau_z(y,z)\big|_x$ — 位置的函数.

显然，轴力的来源为 σ_x，i.e., $N_x(x) = \int_A \sigma_x(y,z)\, dA$



同样，剪力 $Q_y = \int_A \tau_y\, dA$，$Q_z = \int_A \tau_z\, dA$

扭矩 $M_x = \int_A (\tau_z y - \tau_y z)\, dA$

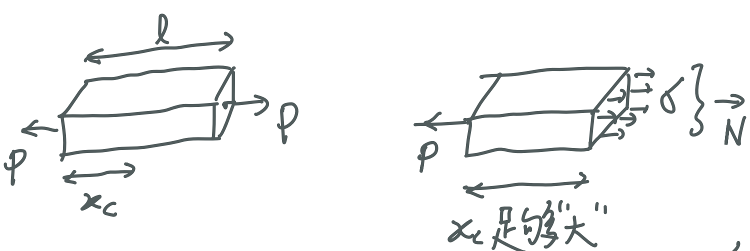


$M_y = \int_A \sigma_x z\, dA$

$M_z = \int_A -\sigma_x y\, dA$

让我们通过最为简单的均匀应力状态来理解应力的概念，i.e. $\sigma_x, \tau_y, \tau_z = C$.

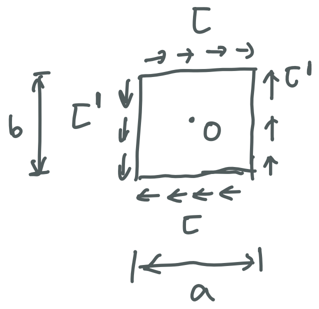
- 纯拉伸



$N(x = x_c) = \sigma \cdot A = P$

σ 比 N 更能反应材料的受力状态.

·纯剪切

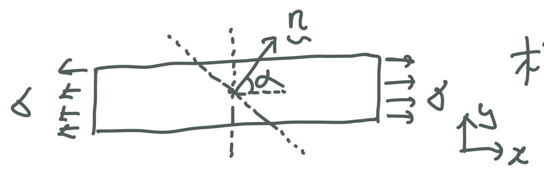


$\Sigma M_o = 0 \rightarrow b \cdot \tau a = a \cdot \tau' \cdot b$

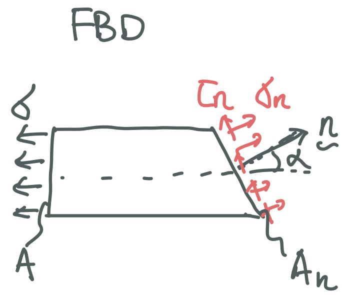
i.e., $\tau = \tau'$ 切应力互等.

a, b 为任意，可为 dy, dz

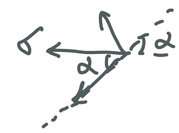
·纯拉伸斜截面



根据应力的标示约定



认为 τ_n，σ_n 是只依赖于 α 的常数，根据平衡关系



$$\Sigma F_n = 0 \rightarrow \sigma \cdot A \cdot \cos\alpha = \sigma_n \cdot A_n$$
$$A_n = A/\cos\alpha$$
$$\rightarrow \sigma_n = \sigma\cos^2\alpha$$

$$\Sigma F_\tau = 0 \rightarrow \sigma A \cdot \sin\alpha + \tau_n \cdot A_n = 0 \rightarrow \tau_n = -\frac{1}{2}\sigma\sin 2\alpha$$

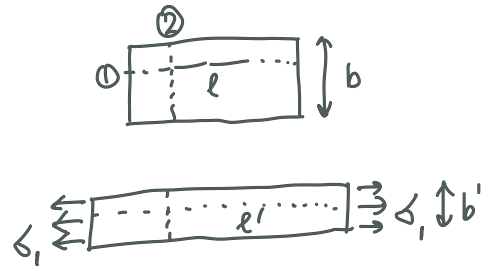
截面上物质点的应力依赖于截面的法向方向。一旦我们知道在一个特定方向的应力，可以通过 FBD 受力分析计算在其它方向的应力水平。为什么关注不同方向呢？材料对 τ 和 σ 的承受能力通常不同。

§1.4 变形、应变、线弹性材料

在外力下，材料会产生变形 (deformation)，i.e.，形状和大小的变化。在小变形下，

应变可以很好的表征变形。我们仍然通过简单应力状态来理解。

• 均匀拉伸



线段①长度变化为 $\Delta l = l' - l$

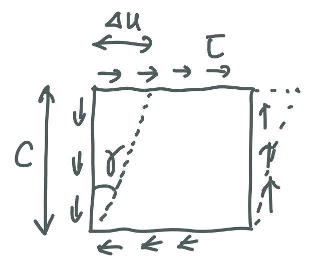
线应变、正应变 $\varepsilon_1 = \frac{\Delta l}{l}$

线段②长度变化为 $\Delta b = b' - b$

横向正应变 $\varepsilon_2 = \frac{\Delta b}{b}$

$\varepsilon_1, \varepsilon_2$ 为常数

• 纯剪切

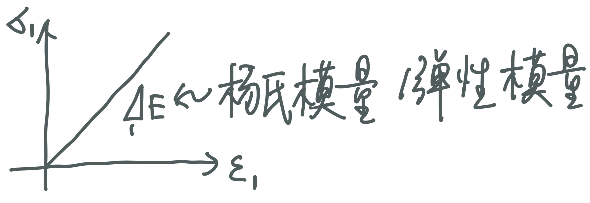


切应变 $\gamma = \tan^{-1}\left(\frac{\Delta u}{c}\right) \simeq \frac{\Delta u}{c}$

后续我们会发现应变与应力一样，逐点"连续"变化，$\varepsilon = \varepsilon(x, y, z)$。此外，应力-应变关系被称为本构关系。我们先关注最为简单的线弹性关系——胡克定律

Hooke's law

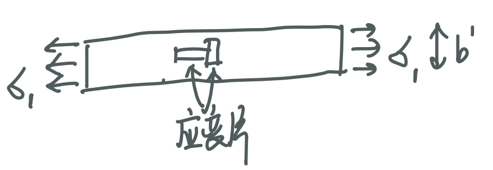
• 简单拉伸



$\varepsilon_1 = \frac{1}{E}\sigma_1$

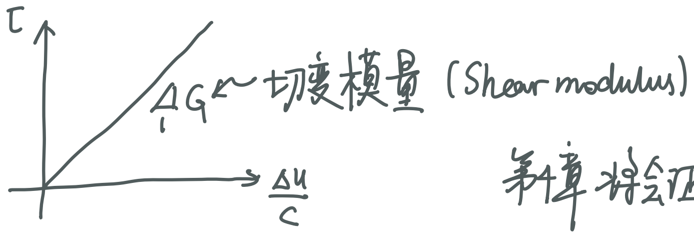
定义 $\frac{\varepsilon_2}{\varepsilon_1} = -\nu$ ← 泊松比（横向变形系数）

$\varepsilon_2 = -\frac{\nu}{E}\sigma_1$



软钢 $E \sim 200$ GPa $\nu \sim 0.28$；软组织 $E \sim 1$ MPa，$\nu \to 0.5$

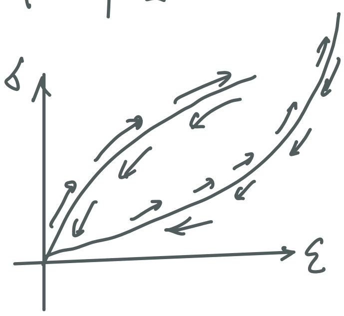
· 纯剪切



$\gamma = \frac{\tau}{G}$

第4章将会证明 $G = \frac{E}{2(1+\nu)}$

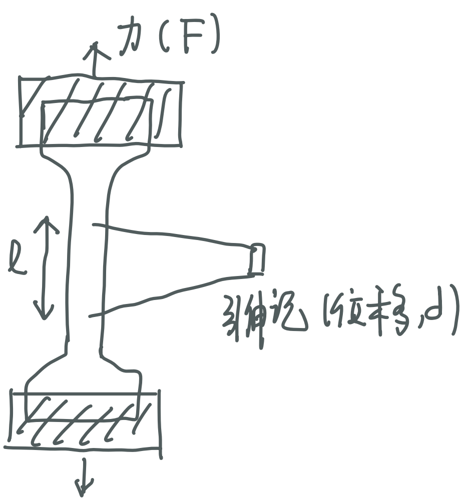
· 非线性弹性

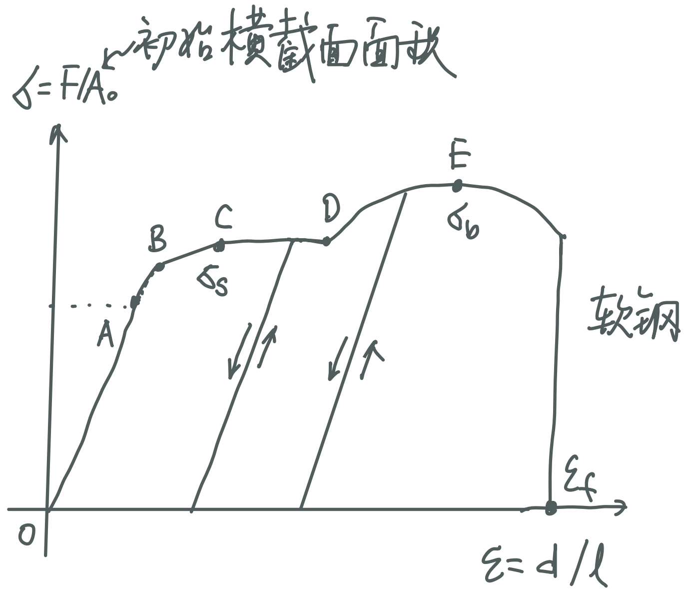


更为一般的本构关系

$\underset{\sim}{\sigma} = f(\underset{\sim}{\varepsilon})$

§1.5 弹塑性材料



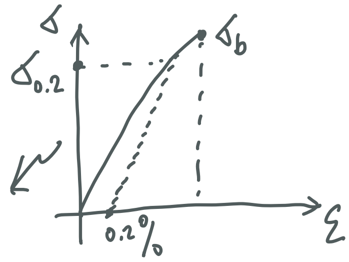


$O \rightarrow A$: 比例极限 $\rightarrow$ B: 弹性极限 $\rightarrow$ C: 屈服应力 $\rightarrow$ E: 强度极限

线弹性 ($O \rightarrow A$)　非线性弹性 ($A \rightarrow B$)　塑性 ($B \rightarrow E$)

$\delta = \varepsilon_f \times 100\% \rightarrow$ $\delta > 5\%$ 韧性 (ductile) 材料

$\delta < 5\%$ 脆性 (brittle) 材料.



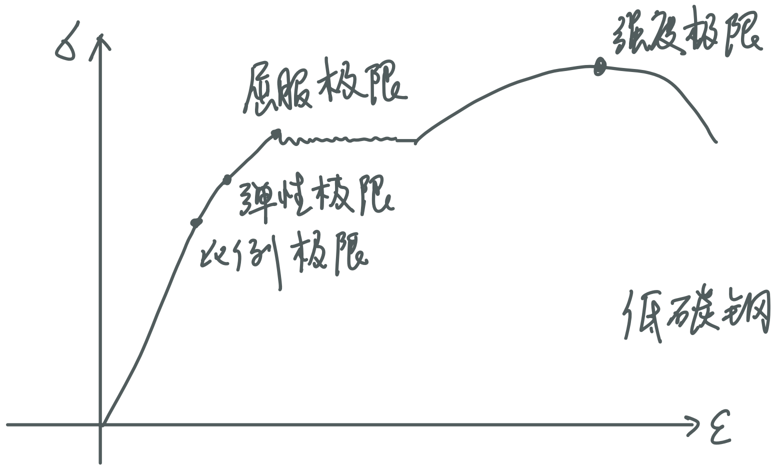
软钢: $\delta \sim 20\text{-}30\%$, $\sigma_s \sim 235\ \mathrm{MPa} \sim \frac{E}{1000}$, $\sigma_b \sim 380\ \mathrm{MPa}$

铸铁: $E \sim 100\ \mathrm{GPa}$, $\delta \sim 1\%$, $\sigma_s = \sigma_{0.2} \sim \frac{2E}{1000}$, $\sigma_b \sim 100\text{-}400\ \mathrm{MPa}$

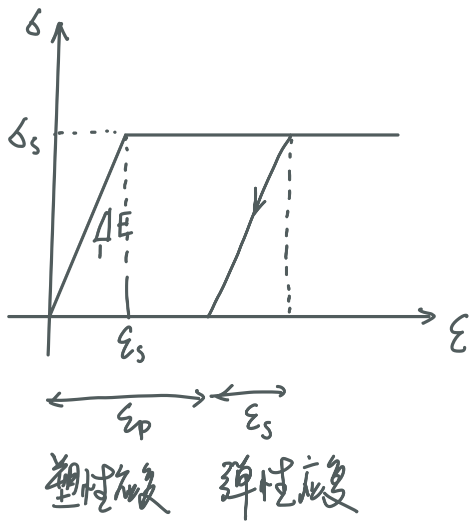
此外, 还有时间相关的黏弹性, 黏塑性, 多孔弹性材料等...

§1.6. 补充

① 弹塑性材料



* 理想弹塑性模型 (id)

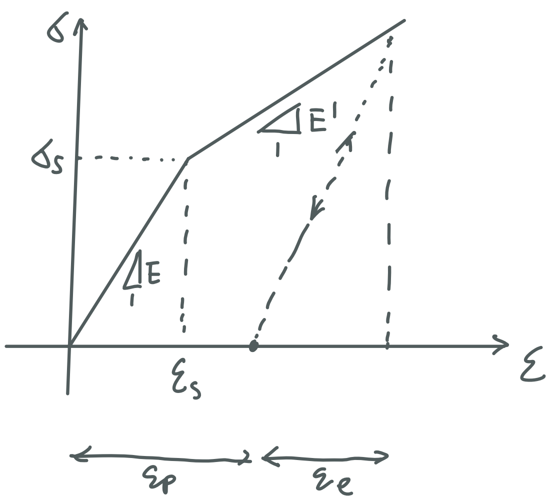


应力-应变曲线

$$\sigma = \begin{cases} E\varepsilon, & \varepsilon \le \varepsilon_s \\ \sigma_s, & \varepsilon > \varepsilon_s \end{cases}$$

$$\varepsilon_s = \sigma_s / E$$

* 线性硬化弹塑性模型



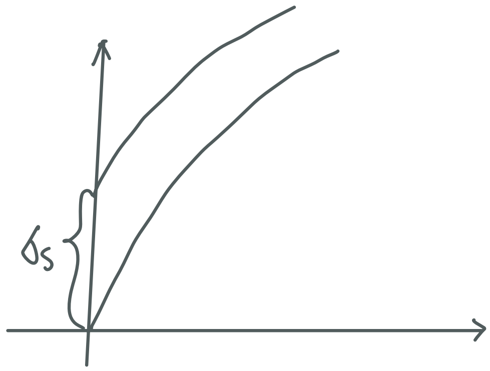
$$\sigma = \begin{cases} E\varepsilon, & \varepsilon \le \varepsilon_s \\ \sigma_s + E'(\varepsilon - \varepsilon_s), & \varepsilon > \varepsilon_s \end{cases}$$

强化模量

*幂次强化模型

$\sigma = K\varepsilon^n$ 或 $\sigma = \sigma_s + K\varepsilon^n$ (Ludwik-Hollomon 公式)

(数学上容易处理)

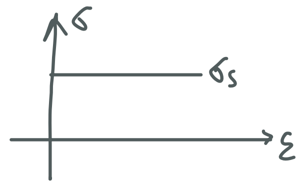


注: ① $K >$, $0 \le n < 1$

② $\frac{d\sigma}{d\varepsilon} = nK\varepsilon^{n-1} \to \infty$ as $\varepsilon \to 0$ (与试验不符

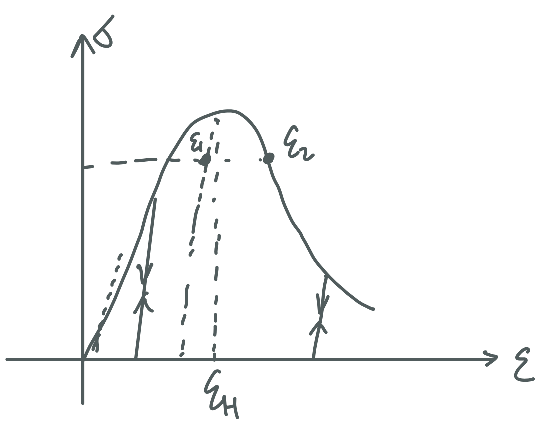
③ $\sigma \to \infty$ as $\varepsilon \to \infty$ (实验会应力饱和)

④ $n \to 0$



理想塑性.

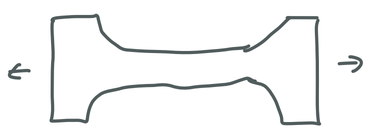
*强化-软化模型



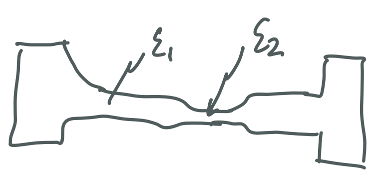
$\sigma = E\varepsilon e^{-\varepsilon/\varepsilon_H}$

· $\left.\frac{d\sigma}{d\varepsilon}\right|_{\varepsilon=0} = E$ (初始切模量)

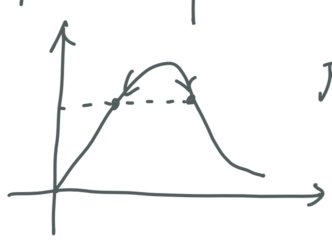
· $\frac{d\sigma}{d\varepsilon} = 0 \to \varepsilon = \varepsilon_H$ (峰值应变)



After ε_H

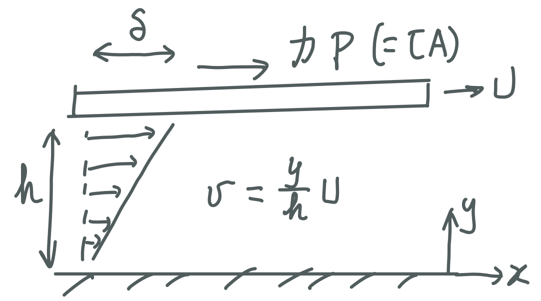


弹性材料



应力-应变沿同一路径返回 (id 相同?)

② 黏弹性材料

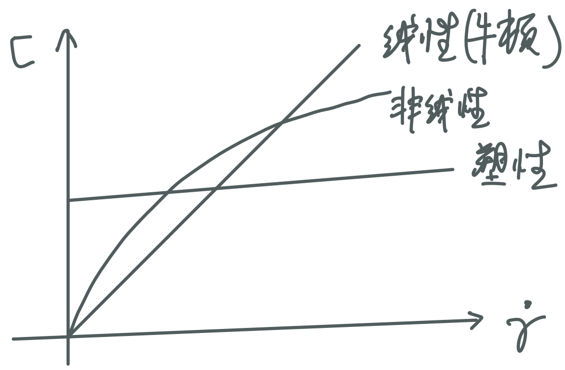


牛顿1687年实验

$\tau = \mu \frac{dv}{dy}$, μ-黏性系数

[Pa·s]

$$\Rightarrow \tau = \mu \frac{d}{dy}\left(\frac{d\delta}{dt}\right) = \mu \frac{d}{dt}\gamma = \mu \dot{\gamma}$$

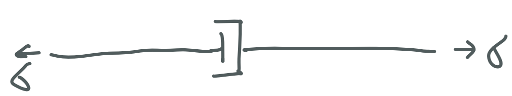


· 乳胶漆 $\dot{\gamma}\uparrow$, $\mu\downarrow$

· 淀粉溶液 $\dot{\gamma}\uparrow$, $\mu\downarrow$

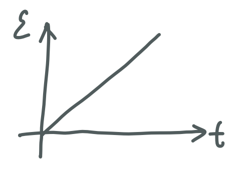
对于拉伸，也有 $\underbrace{\sigma = \mu \dot{\varepsilon}}_{\text{线性黏性本构关系}}$

*(牛顿)黏壶

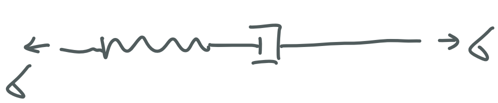


本构关系

$\sigma = \mu \dot{\varepsilon}$



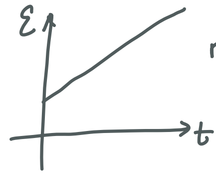
*弹簧与黏壶串联 (Maxwell 模型)



$$\left.\begin{aligned} \sigma &= E\varepsilon_s \\ \sigma &= \mu \dot{\varepsilon}_d \end{aligned}\right\} \ \varepsilon = \varepsilon_s + \varepsilon_d$$

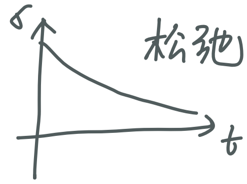
本构关系 $\dot{\varepsilon} = \frac{1}{E}\dot{\sigma} + \frac{1}{\mu}\sigma$

特点：①应力不变，应变随时间增加 (=减小)



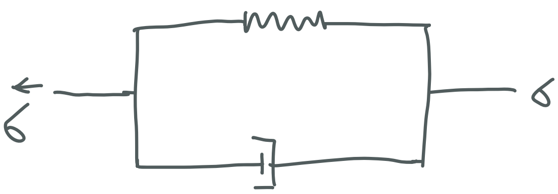
蠕变 教堂玻璃

②应变不变，应力随时间衰减 $\sigma(t) = Ae^{-\frac{E}{\mu}t}$



松弛

* 弹簧与黏壶并联 (Kelvin-Voigt 模型)



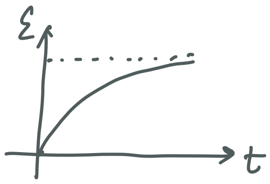
$\sigma_s = E\varepsilon$

$\sigma_d = \mu\dot{\varepsilon}$

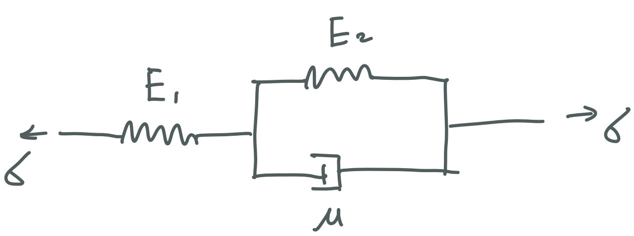
$\sigma = \sigma_s + \sigma_d$

本构关系：$\sigma = E\varepsilon + \mu\dot{\varepsilon}$

特点：$\sigma = \sigma_0$，$\varepsilon = A(1 - e^{-\frac{E}{\mu}t})$



* 标准线性固体模型



$\sigma = E_1\varepsilon_1$

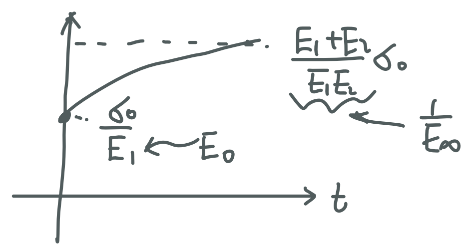
$$\left.\begin{aligned}\sigma_2 &= E_2\varepsilon_2\\ \sigma_d &= \mu\dot{\varepsilon}_2\end{aligned}\right\}\ \sigma = E_2\varepsilon_2 + \mu\dot{\varepsilon}_2$$

$\sigma = E_2(\varepsilon - \varepsilon_1) + \mu(\dot{\varepsilon} - \dot{\varepsilon}_1)$

$$\sigma = E_2\varepsilon - \frac{E_2}{E_1}\sigma + \mu\dot{\varepsilon} - \mu\frac{\dot{\sigma}}{E_1}$$

$$\Rightarrow \quad E_1E_2\varepsilon + E_1\mu\dot{\varepsilon} = (E_1+E_2)\sigma + \mu\dot{\sigma}$$

特点：① 阶跃应力 $\sigma=\sigma_0$，$\varepsilon(t) = \frac{\sigma_0}{E_1} + \frac{\sigma_0}{E_2}\left(1-e^{-\frac{t}{\tau_\sigma}}\right)$，$\tau_\sigma = \mu/E_2$



② 阶跃应变 $\varepsilon=\varepsilon_0$，$\sigma(t) = E_1\varepsilon_0 - \frac{E_1^2\varepsilon_0}{E_1+E_2}\left(1-e^{-\frac{t}{\tau_\varepsilon}}\right)$，$\tau_\varepsilon = \frac{\eta}{E_1+E_2}$