断裂力学

Fracture Mechanics

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College of Engineering, Peking University

19 February 2024

Outline

I. About this course

II. Why I wanted to develop this course

III. Why fracture mechanics

- ❑ History of fracture mechanics
- ❑ Some scenarios in nature and engineering systems

IV. Back to the course

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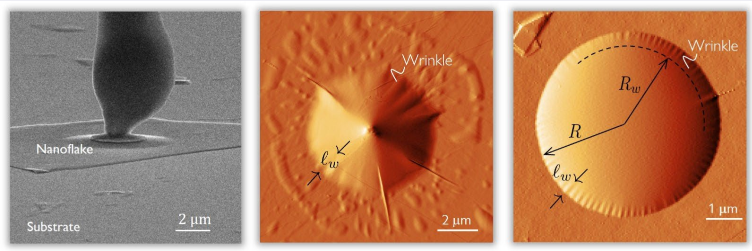
Group: 断裂力学 2024

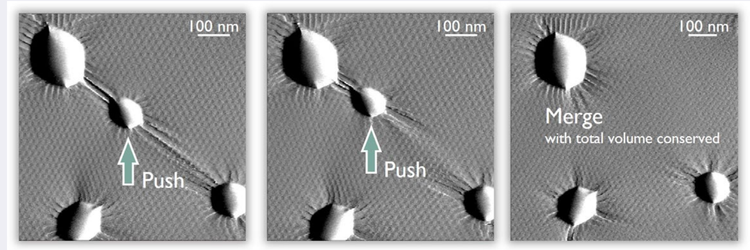
Valid until 2/24 and will update upon joining group

The Instructor

- ❑ B.S. from USTC
- ❑ M.S. from Institute of Mechanics
- ❑ Ph.D. from University of Texas at Austin
- ❑ Post-doc at Oxford University

Keywords: Thin film mechanics, 2D materials, Adhesion, Friction, Wetting



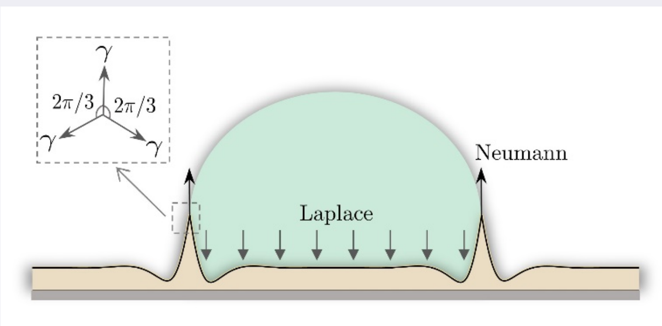


Thin film mechanics
Nonlinear elasticity, wrinkling, FvK

J. Appl. Mech. (2023)
Nano Lett. (2023)
JMPS (2021)
Phys. Rev. Lett. (2019)
Adv. Funct. Mater. (2016)

Surfaces and interfaces (solids)
Adhesion, friction, fracture

JMPS(2023); IJSS (2022)
JMPS (2020)
Curr. Opin. Solid State Mater. Sci. (2020)
Adv. Mater. (2019); Carbon (2019)
Phys. Rev. Lett. (2018); Phys. Rev. Lett. (2017)
Composites Sci. Tech. (2016)



Surfaces and interfaces (liquids)
Wetting, liquid-solid interactions

Sci. Adv. (2023)
Phys. Rev. Fluids (2022)
JMPS (2021)
Nat. Commun. (2021)
PNAS (2018)

Self-Introduction

- ❑ Your name & background
- ❑ Your department & adviser
- ❑ Your research topics
- ❑ Why study fracture mechanics

Introduction of the course

开课教师:	戴兆贺	**邮件:**	daizh@pku.edu.cn	**时间:**	M(Odd weeks) 8–10 am; W 1–3 pm
教室:	二教 313	**课程号:**	00334510	**学分:**	3

Course Description: Fracture mechanics is the field of mechanics concerned with the study of the propagation of cracks in materials. The processes of material manufacture, processing, machining, and forming may introduce flaws in a finished mechanical component. Through the manufacturing process, interior and surface flaws are found in all metal structures. Not all such flaws are unstable under service conditions. Fracture mechanics is the analysis of flaws to discover those that are safe (that is, do not grow) and those that are liable to propagate as cracks and so cause failure of the flawed structure. Fracture mechanics as a subject for critical study has barely been around for a century and thus is relatively new.

This course will emphasize the basic principles that underlie the mechanics of fracture. The principle components of a fracture mechanics problem include the determination of the stress and strain fields in a cracked body and a fracture criterion for crack advance. In this course we will mainly cover linear fracture mechanics concepts and look at separation mechanisms near a crack tip. A few nonlinear fracture topics will also be discussed.

Prerequisites: Calculus, differential equations, linear algebra, mechanics of materials, and elasticity.

Course Outline

- ❑ T1: Ideal strength and Griffith theory
- ❑ T2: Energy release rate
- ❑ T3: Stress intensity factors
- ❑ T4: Westergaard's stress functions
- ❑ T5: Weight function method
- ❑ T6: Cyclic loading, R-curve, Mixed mode loading
- ❑ T7: Dugdale-Barenblatt model
- ❑ T8: J integral
- ❑ T9: Peeling, tearing, buckle delamination
- ❑ T10: Adhesion theories
- ❑ T11: Dynamic fracture mechanics
- ❑ T12: Interfacial fracture mechanics
- ❑ T13: Anisotropic crack tip field
- ❑ T14: Atomic origin of surface energies

References

References: Unfortunately, there is no single comprehensive text for this course. The skills and information required to complete the problem sets are primarily contained in the lectures. However, additional references and perspectives are always useful for those who want to explore the covered topic further, so below is a list of useful advanced texts on interfacial phenomena, solid mechanics, and fluid mechanics:

- K.B. Broberg, *Cracks and Fracture*, Academic Press, 1999. (A good text for a graduate fracture mechanics course)
- D. Maugis, *Contact, adhesion and rupture of elastic solids*, Springer, 2013 (Great contents on adhesion problems)
- C. Landis, *Lecture notes for fracture mechanics* (from who I learnt this course)
- J.R. Rice, *Mathematical Analysis in the Mechanics of Fracture*, in Fracture: An Advanced Treatise, vol. 2, ed. by H. Liebowitz, Academic Press, 1968. (Excellent treatment of the J-integral, cohesive zone models, and other topics in linear and nonlinear fracture)

Fourier transforms by I. N. Sneddon; Foundations of applied mathematics by M. D. Greenberg; Mixed boundary value problems by D. G. Duffy

Grades and dates

Evaluation method:

Homework	$15\% \times 4 = 60\%$
Midterm 1	15%
Midterm 2	15%
Project presentation	10%
TOTAL	100%

Important Dates:

Homework	Every 7–14 days
Midterm 1	Around week 7
Midterm 2	Around Week 13
Final Presentation	Week 16

Policies

Homework Policy: Problem sets will be assigned every 7-14 days. Complete the set of problems in time because lectures and homework build on previous homework. Each problem is worth 10 points and is graded on a scale of 0 to 10. Homework may be handed in late without penalty only if prior arrangements have been made with the instructor and there are extenuating circumstances. Homework is meant as an exercise and you are encouraged to consult anyone (other students, me) and anything (notes, books) if it helps you to understand and learn the material. Discussions with other students in the class are especially encouraged. However, you may not consult another person's solution to any given problem.

Test Policy: Two "tests" will be given in this course. The test will be exactly like the homework. You will be allowed to consult your own notes from class and the textbooks approved by me.

Project Policy: Students can either devise and solve an illustrative problem in fracture mechanics or solve a relevant, 'interesting' problem reported in the literature. The problem should be comparable in scope and difficulty to the more challenging homework problems given in this course. Each student will give a 10-minute presentation plus a 2-minute Q&A to the class at the end of the semester.

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The philosophy of the course

Generative AI is experimental. Info quality may vary.

A scientist is a professional who conducts research to further knowledge in a particular area.

- ❑ They make observations in nature and conduct experiments to test their observations.
- ❑ They use scientific methods to explain the natural world.
- ❑ They believe that there is a natural explanation for most things.

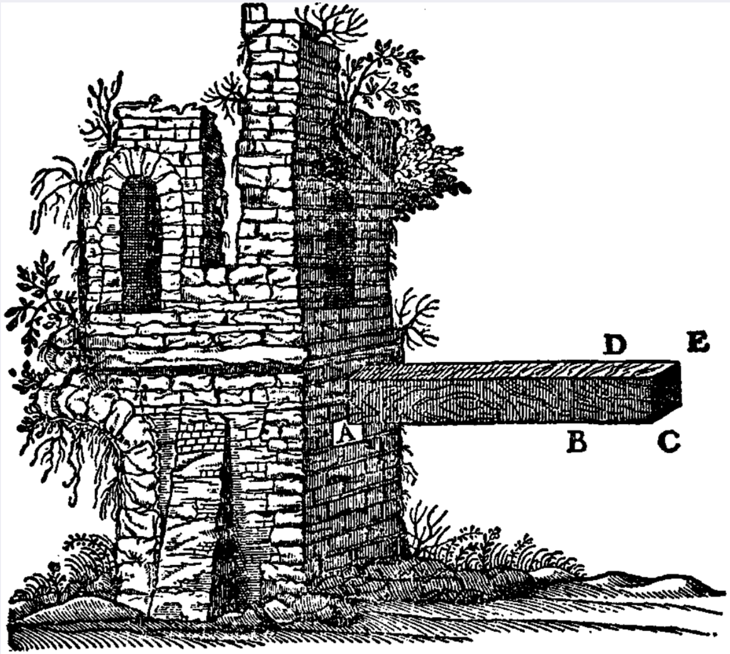
Galileo Galilei (1638)

Not only new knowledge but also the process by which it is created!

A famous "fracture mechanics" problem

Dante Alighieri's vision of hell, which Galileo attempted to map.

伽利略早期的方法和结论



Dome: ~45 meters in diameter and ~3 meters in thickness

Hell: ~45 km in radius (from Jerusalem to Marseille) and thus ~**6 km** in thickness

“On the Shape, Location, and Size of Dante's Inferno”, Galileo Galilei (1588)

Make further observations

SALV. For a while, Simplicio, I used to think, as you do, that the resistances of similar solids were similar; but a certain casual observation showed me that similar solids do not exhibit a strength which is proportional to their size, the larger ones being less fitted to undergo rough usage just as tall men are more apt than small children to be injured by a fall. And, as we remarked at the outset, a large beam or column falling from a

[165]

given height will go to pieces when under the same circumstances a small scantling or small marble cylinder will not break. It was this observation which led me to the investigation of the fact which I am about to demonstrate to you: it is a very remarkable thing that, among the infinite variety of solids which are similar one to another, there are no two of which the forces [*momenti*], and the resistances of these solids are related in the same ratio.

Galileo Galilei (1638): Proposition IV of Day 2

Apply scientific methods

to be devoid of weight. But if the weight of the prism is to be taken account of in conjunction with the weight E, we must add to the weight E one half that of the prism BD: so that if, for example, the latter weighs two pounds and the weight E is ten pounds we must treat the weight E as if it were eleven pounds.

SIMP. Why not twelve?

SALV. The weight E, my dear Simplicio, hanging at the extreme end C acts upon the lever BC with its full moment of ten pounds: so also would the solid BD if sus-

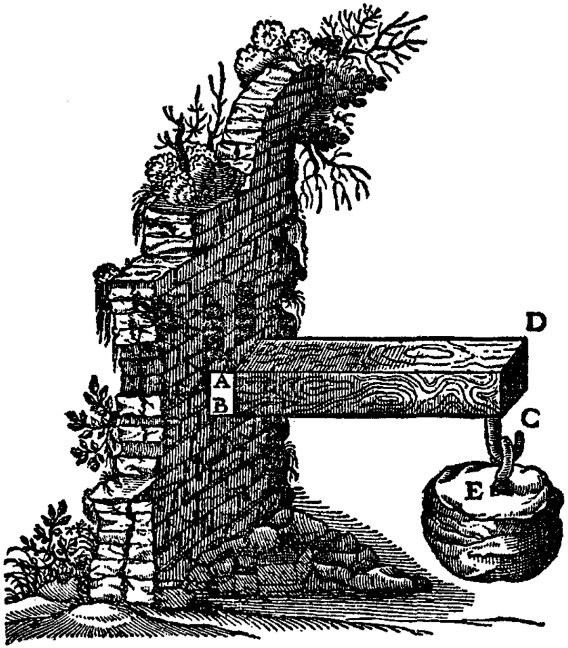


Fig. 17

Galileo Galilei (1638): Proposition IV-VIII of Day 2

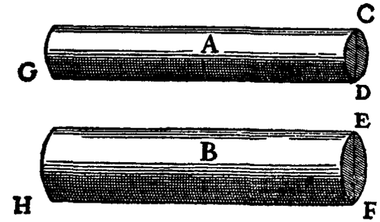


Fig. 20

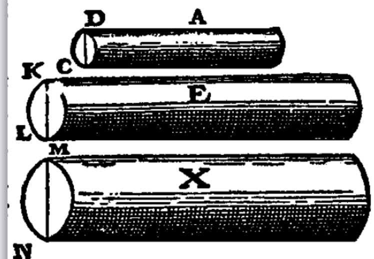


Fig. 26

- ❑ Galileo's square-cube Law
- ❑ CD:KL=KL:MN

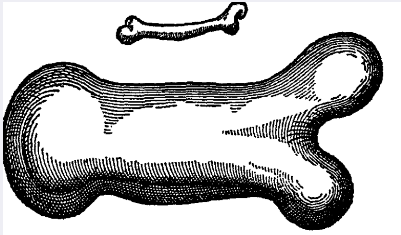
Draw new knowledges

Dome: ~45 meters in diameter and ~3 meters in thickness

Hell: ~ 45 km in radius (from Jerusalem to Marseille) and thus ~**6km*2000** in thickness

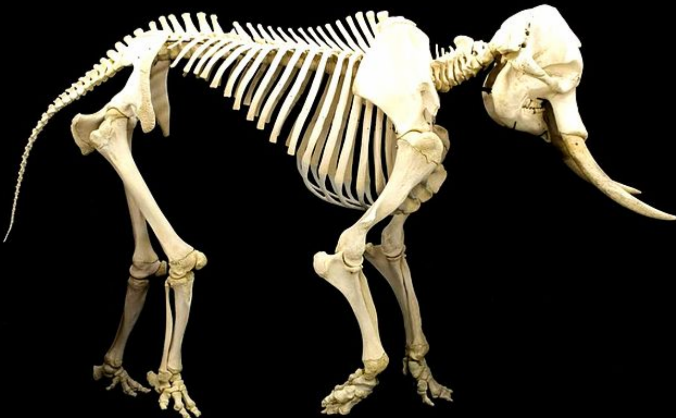
From what has already been demonstrated, you can plainly see the impossibility of increasing the size of structures to vast dimensions either in art or in nature; likewise the impossibility of building ships, palaces, or temples of enormous size in such a way that their oars, yards, beams, iron-bolts, and, in short, all their other parts will hold together; nor can nature produce trees of extraordinary size because the branches would break down under their own weight; so also it would be impossible to build up the bony structures of men, horses, or other animals so as to hold together and perform their normal functions if these animals were to be increased enormously in height; for this increase in height can be accomplished only by employing a material which is harder and stronger than usual, or by enlarging the size of the bones, thus changing their shape until the form and appearance of the animals suggest a monstrosity. This is

"The roof would have to be so thick that there would hardly be any room underneath to accommodate all those dead souls."

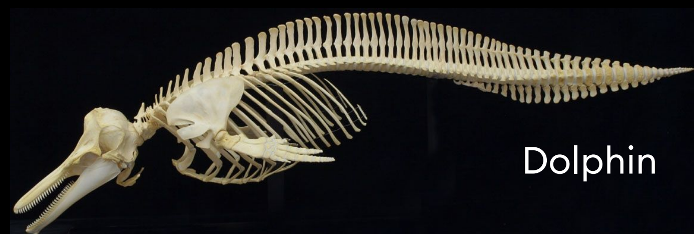


Scaling

Elephant



Rat



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Giant's Causeway

- **2 Chainz: No matter where I'm at, I got crack**

- **Wolfgang Pauli: God made the bulk; the surface was invented by the devil.**

- **José Bico: 断裂让科学着迷，因为他触及了物质内在的性质。你可曾意识到撕纸时你用手打破了原子之间的联系？原子们被分在了两边。**

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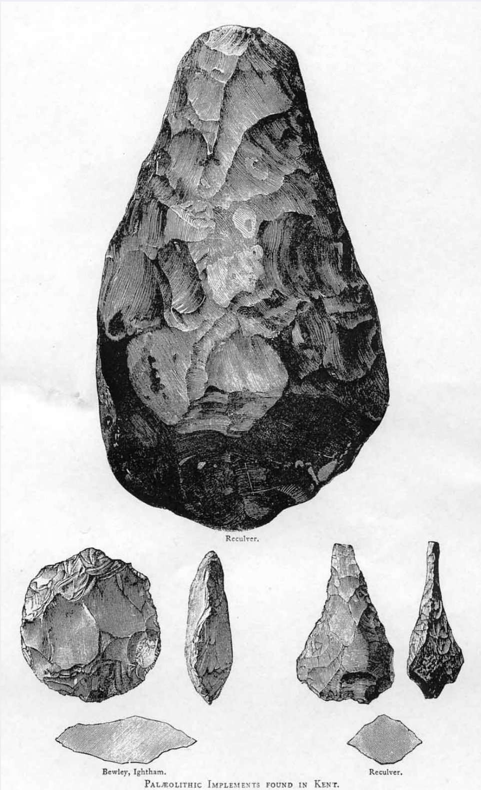
Shaping brittle materials by fractures using knowledge based on experience



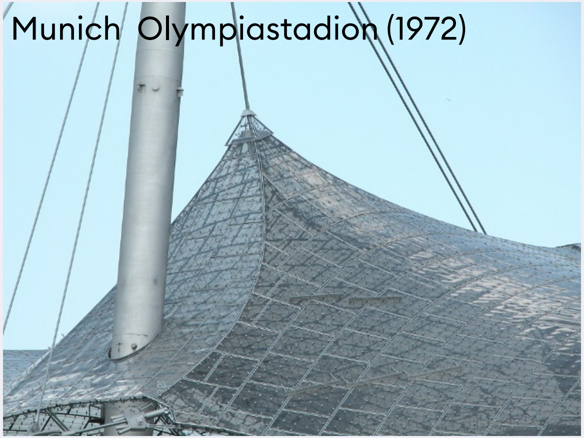
Acheulean hand-axes from Kent

Skull cup from the Gough's Cave.
Photograph: Derek Adams

From woods and stones to steels and glasses


Cathedral of Santa Maria del Fiore (1436)

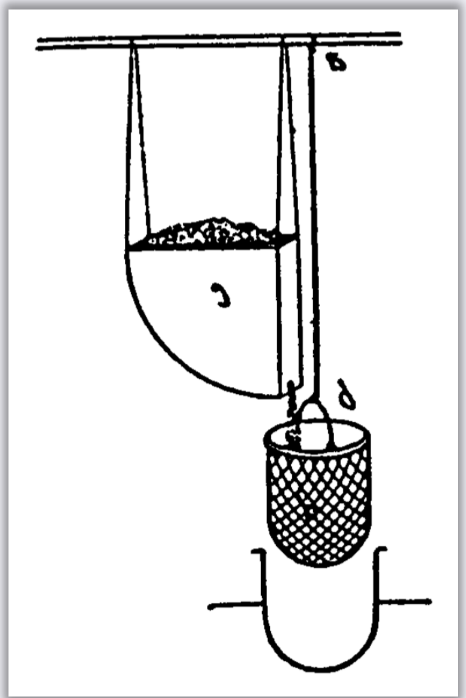

赵州桥 (605)


Munich Olympiastadion (1972)

Leonardo da Vinci's fracture test (1452–1519)

The object of this test is to find the load an iron wire can carry.

- Attach an iron wire to a basket
- Feed into the basket some fine sand through a small hole placed at the end of a hopper.
- A spring will close the hole as soon as the wire breaks.
- The basket falls a short distance into a hole, so as not to upset the basket.
- The weight of the sand and the location of fracture of the wire are to be recorded.
- The test is repeated several times to check the results.
- Then a wire of one-half the previous length is tested and the additional weight it carries is recorded;
- the a wire of one fourth length is tested and so forth,



The strength of a metal wire increases with decreasing length.
This size effect is the result of the decreasing number of which were clearly visible in metal wires at that time.

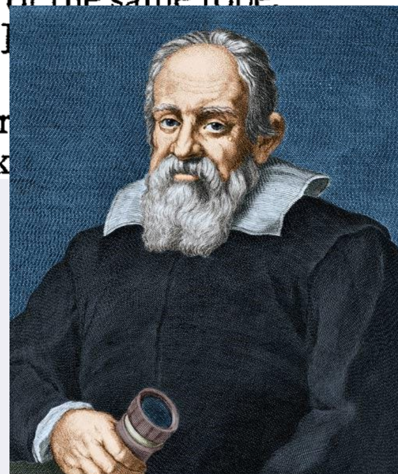
Galileo Galilei (1564–1642)

Day two

SALV. I fear, Simplicio, if I correctly catch your meaning, that in this particular you are making the same mistake as many others; that is if you mean to say that a long rope, one of perhaps 40 cubits, cannot hold up so great a weight as a shorter length, say one or two cubits, of the same rope.

SIMP. That is what I … ee the proposition is highly probable.

SALV. On the contr… erely improbable but false; and I think … ou of your error.



the short one? Give up then this erroneous view which you share with many very intelligent people, and let us proceed.

In 19th century, elasticity theory is developed, with some criteria for failure and fracture

- Coulomb (1763–1806): Ultimate shear strength (stress/strain not clearly defined)
- Saint Venant (1797–1886): Maximum strain hypothesis (until the end of the 19th century)
- Lamé (1795–1870), Rankine (1820–1872): Maximum stress criterion (for brittle materials)
- Tresca (1814–1885): Maximum shear criterion ("for plastic materials")
- Mohr (1835–1913): General failure criterion using Mohr's circles (Mohr-Coulomb criterion, still in use in Geomechanics)

Until the end of the 19th century there was not yet a clear differentiation between failure by brittle fracture and failure by plastic flow

- Huber (1872–1950), von Mises (1883–1953), Hencky (1885–1951): Maximum distortion energy criterion

Good for single components but cannot explain the failure of components of cracks e.g., Boston Molasses Disaster!

From fracture criterion to fracture mechanics

The disadvantage of these criterion:

- ❑ Do not consider the kinematics of failure
- ❑ Do not consider thermodynamic principles (e.g. the energy balance of a failure process)
- ❑ Do not consider the fact that real materials always have defects and cracks.

Mathematics Genealogy Project

Carl Friedrich Daniel Wieghardt

Dr. phil. Georg-August-Universität Göttingen 1903

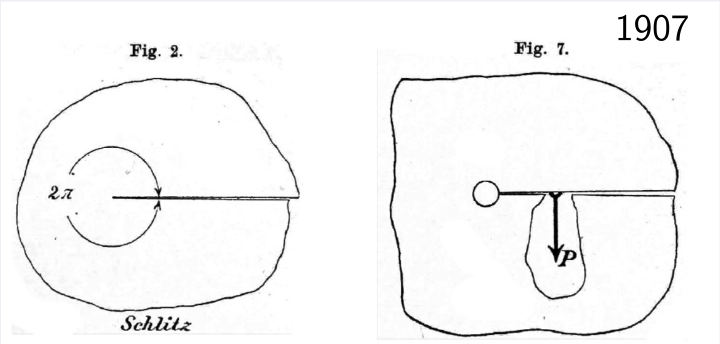
Dissertation: *Über die Statik ebener Fachwerke mit schlaffen Stäben*

Mathematics Subject Classification: 70—Mechanics of particles and systems

Advisor: C. Felix (Christian) Klein

Considering kinematics and defects:

- Found exact solution for the singular crack tip field
- Recognized that the traditional failure criteria cannot predict failure of real cracks.
- Proposed that Stress level in a small area becomes critical
- Did not pursue this problem further…



Griffith's energetic fracture concept

Alan A. Griffith
(1893–1963)

VI. *The Phenomena of Rupture and Flow in Solids.*

By A. A. Griffith, *M. Eng.* (*of the Royal Aircraft Establishment*).

Communicated by G. I. Taylor, *F.R.S.*

Received February 11,—Read February 26, 1920.

The potential energy of the surface of the crack, per unit thickness of the plate is

$$U = 4cT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where T is the surface tension of the material.

Hence the total diminution of the potential energy of the system, due to the presence of the crack, is

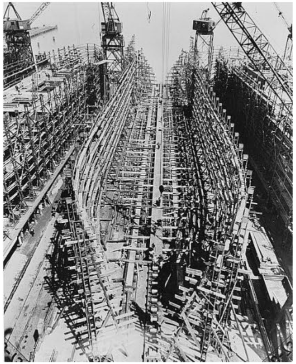
$$W - U = \frac{(3-p)\,\pi c^2 R^2}{8\mu} - 4cT. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The condition that the crack may extend is

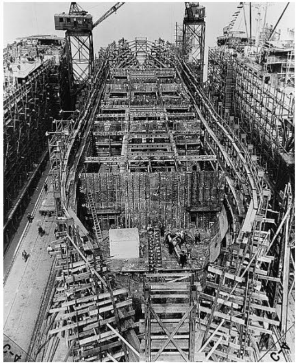
$$\frac{\partial}{\partial c}(W - U) = 0,$$

Recognized and generally accepted by the scientific community, it had no impact on engineering practice: i) The big discrepancies in surface energy for ductile materials; ii) The difficulty to determine the change of strain energy w.r.t. crack growth.

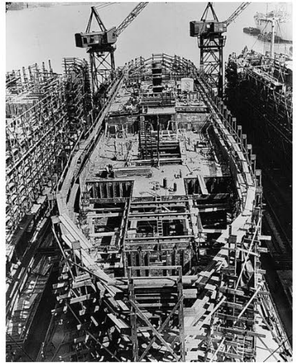
The Liberty ships



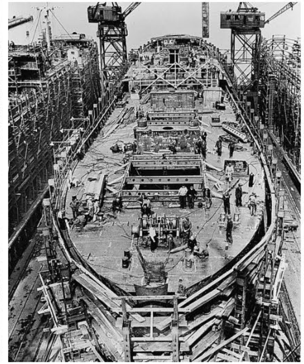
Day 2 : Laying of the keel plates



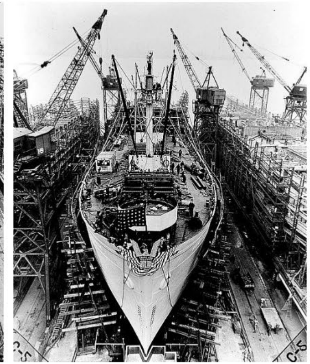
Day 6 : Bulkheads and girders below the second deck are in place.



Day 10 : Lower deck being completed and the upper deck amidship erected



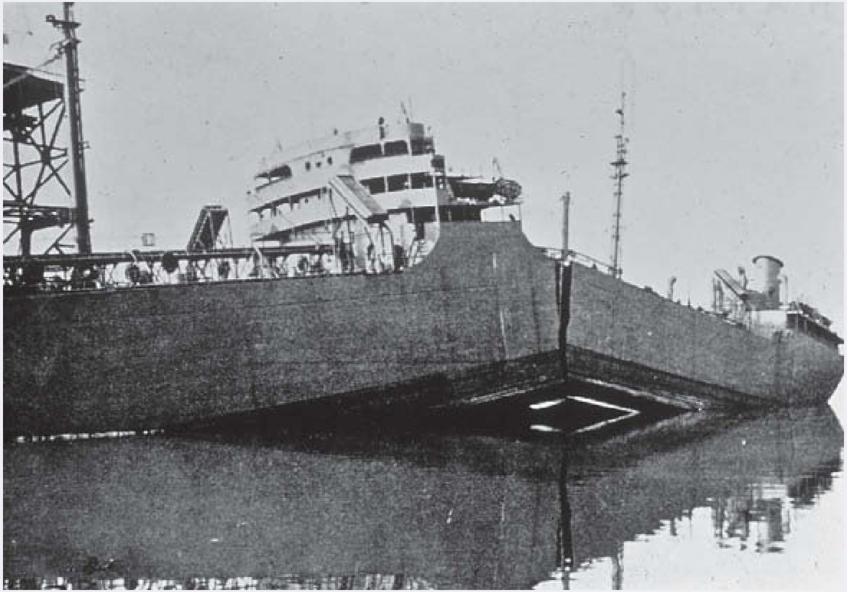
Day 14 : Upper deck erected and mast houses and the after-deck house in place



Day 24 : Ship ready for launching

- Replaced much riveting with welding
- 2,751 Liberties were built between 1941 and 1945 (an average of three ships every two days, symbolizing U.S. wartime industrial output)

The Liberty ships

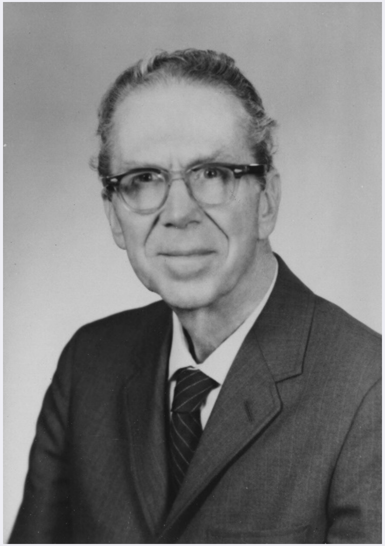


The Liberty ship that was fractured the day after it was launched in 1943

- ❑ As of 1946, 1,031 out of 2,751 damages or accidents due to brittle fractures had been reported. 19 ships breaking in half without warning (behaving like brittle, why?).

- ❑ Reasons: Stress concentrations at the hatch corners, weld defects and fatigue cracks

Irwin's stress intensity factors



George R. Irwin
(1907-1998)
US Naval Research Laboratory

- In 1948, replaced Griffith's surface energy by the effective fracture surface energy (incorporating the work plastic deformation at the crack tip during crack)
- in 1957, using Westergaard's method (1939), pointed out the singular crack tip field which is equivalent with Griffith's theory and measurable.

$$\sigma_y = \left(\frac{E\mathcal{G}}{\pi}\right)^{1/2} \frac{\cos\theta/2}{\sqrt{(2r)}}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \ldots\ldots [10]$$

and

$$\sigma_x = \left(\frac{E\mathcal{G}}{\pi}\right)^{1/2} \frac{\cos\frac{\theta}{2}}{\sqrt{(2r)}}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - \sigma_{ox} \ldots [11]$$

Also found 50 years ago by Wieghardt but forgotten (based on strength criteria since no Griffith's theory yet)

Irwin's fracture concept found rapidly entrance into practical applications and is meanwhile firmly established. Then elastic-plastic fracture mechanics, fatigue, damage mechanics, J integral (Ductile Fracture Handbook by EPRI)

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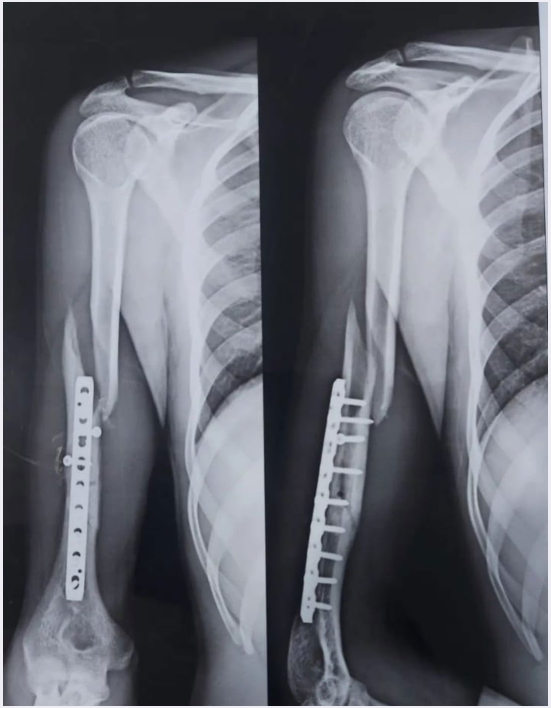
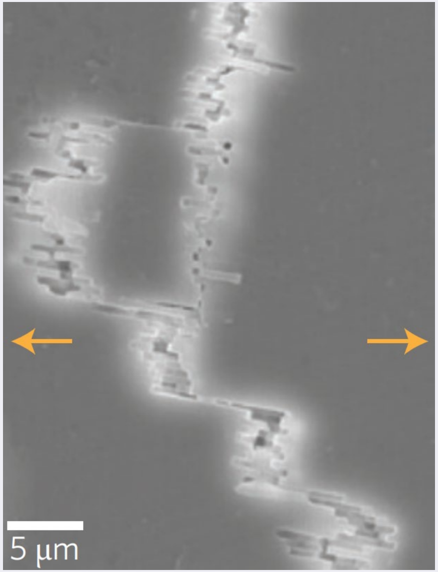
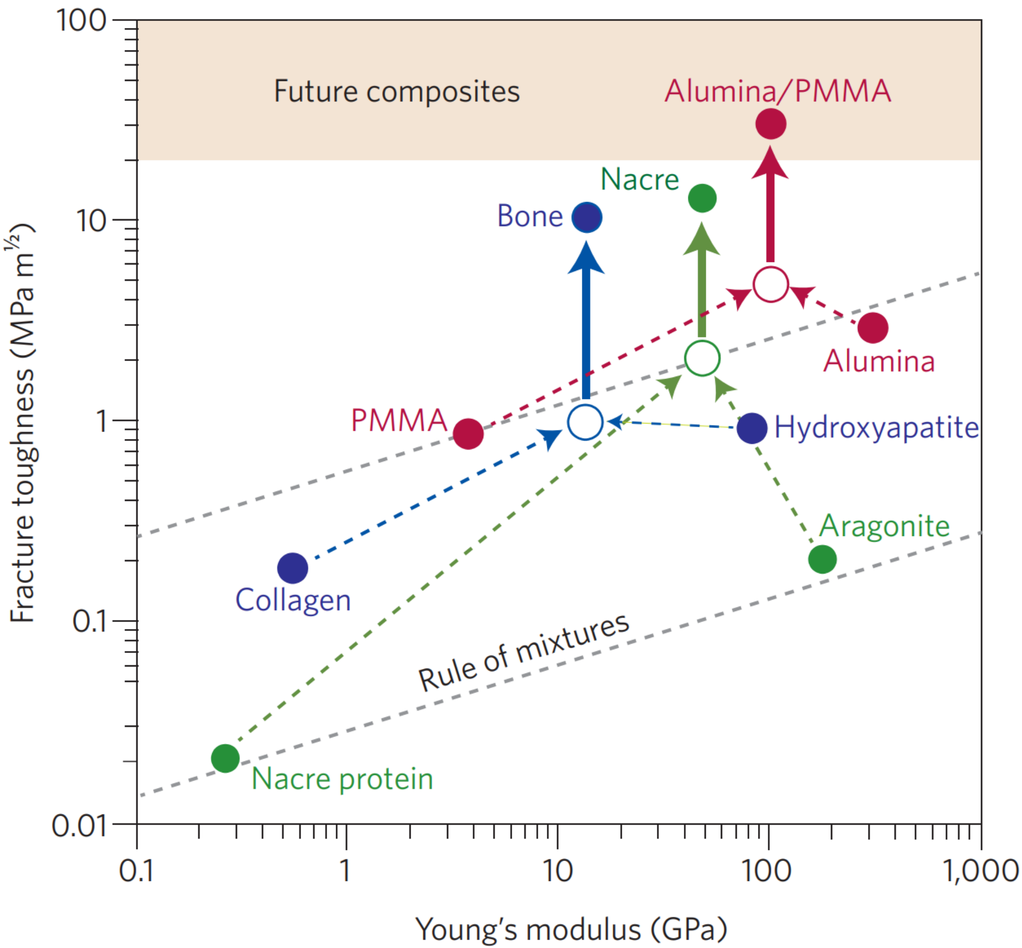
1. Tough biomaterials and composites

Wegst et al. Nat. Mater. 2014



Bone broken by arm wrestling

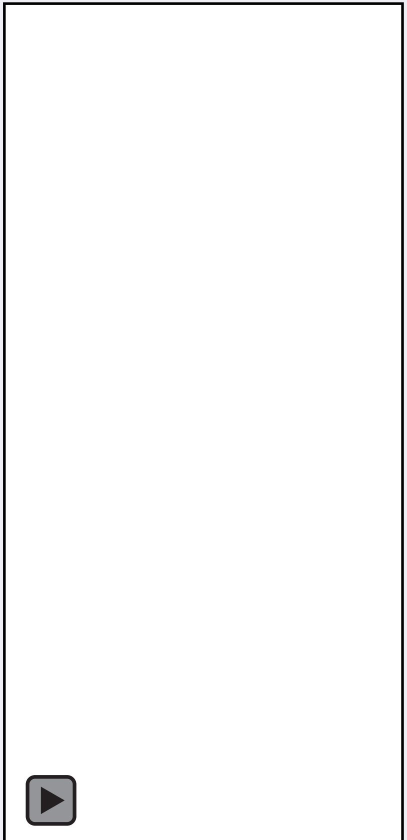




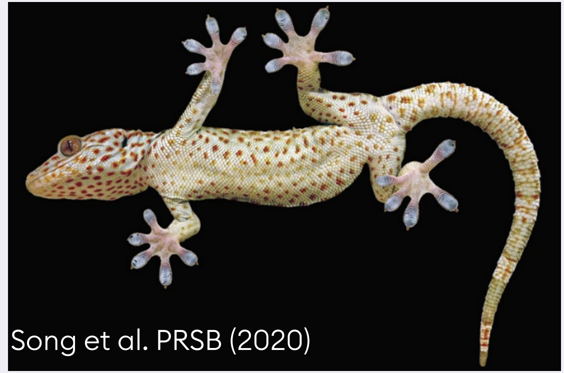
Future composites
Alumina/PMMA
Nacre
Bone
Alumina
PMMA
Hydroxyapatite
Aragonite
Collagen
Rule of mixtures
Nacre protein
Fracture toughness (MPa m½)
Young's modulus (GPa)
100
10
1
0.1
0.01
0.1
1
10
100
1,000

2. Dry adhesives

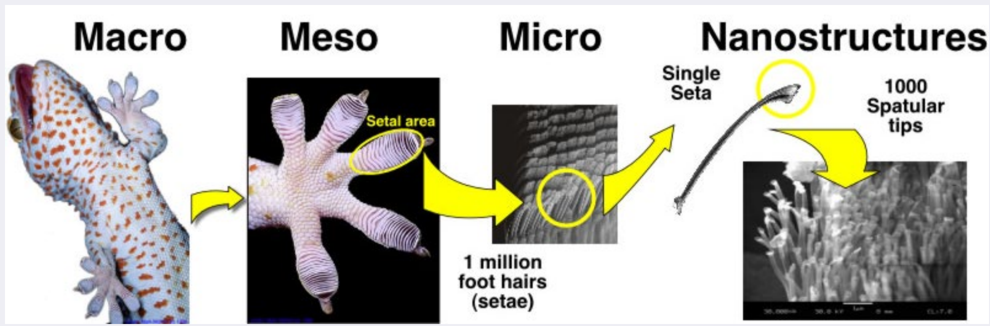
CHROMOS Industrial (YTB)



How gecko's stick to walls: A natural application of van der Waals forces



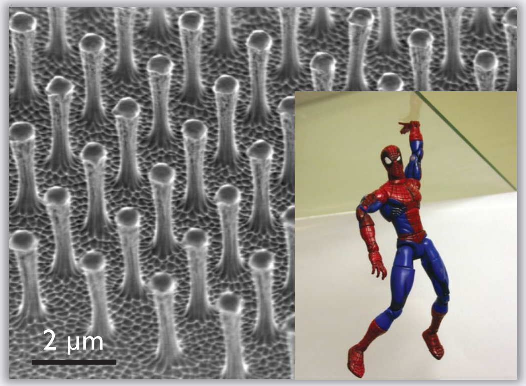
Song et al. PRSB (2020)



Gecko Project (Berkely)

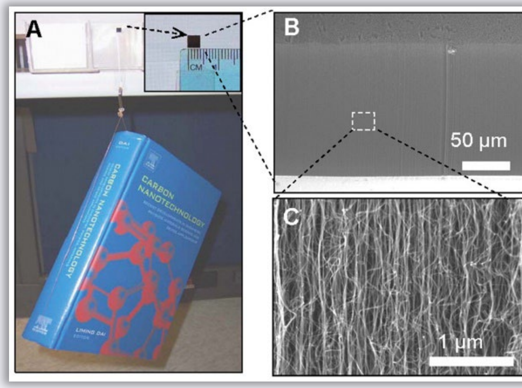
Related course lecture: van der Waals forces

Array of polyimide hairs

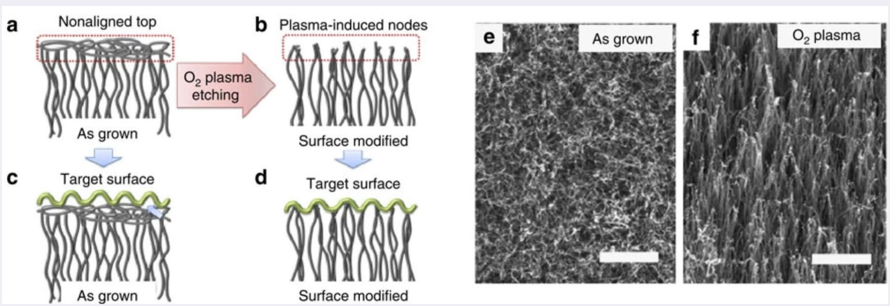


Geim et al. Nat. Mater. (2003)

Carbon nanotube arrays

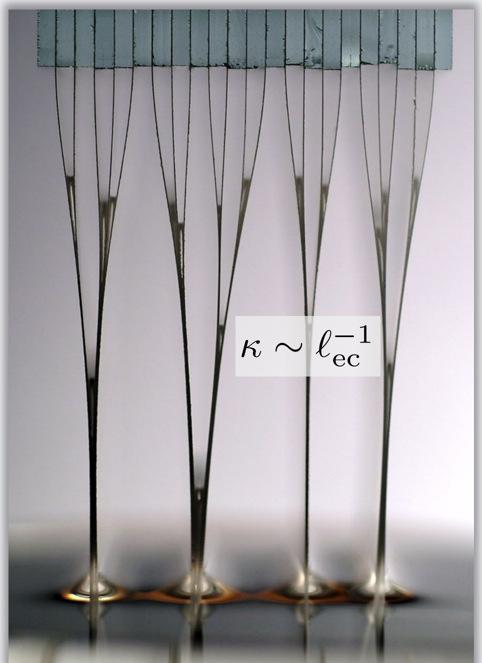


Qu et al. Science (2008)



Xu et al. Nat. Commun. (2016)

3. Wet adhesion



- ❑ Smaller, “stronger”

$$\frac{\gamma \times \mathrm{area}}{U_{\mathrm{e}} \times \mathrm{volume}} \sim \frac{\gamma}{U_{\mathrm{e}}} \times \frac{1}{t}$$

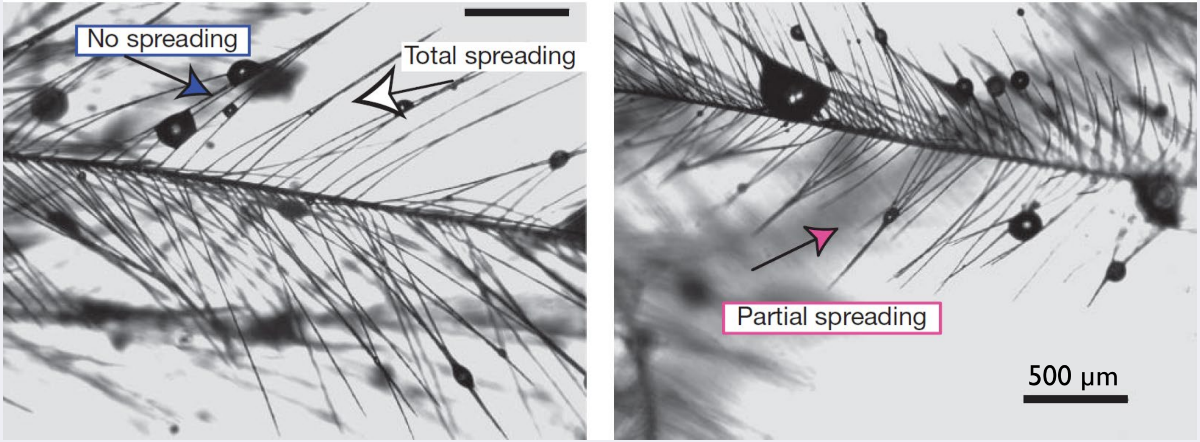
- ❑ What is meant by “strong”?

$$\ell_{\mathrm{ec}} \sim \left(\frac{B}{\gamma}\right)^{1/2}$$

An elastocapillary length-scale

Bico et al. Nature (2004)

Related course lecture: Energy release rate



Duprat et al. Nature (2012)

One of reasons for why most heave losses of birds due to oil pollution happen in winter.

Hartung, R. J. Wild. Mgmt (1967)

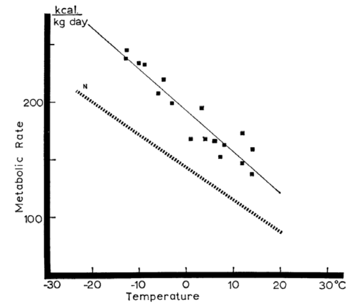
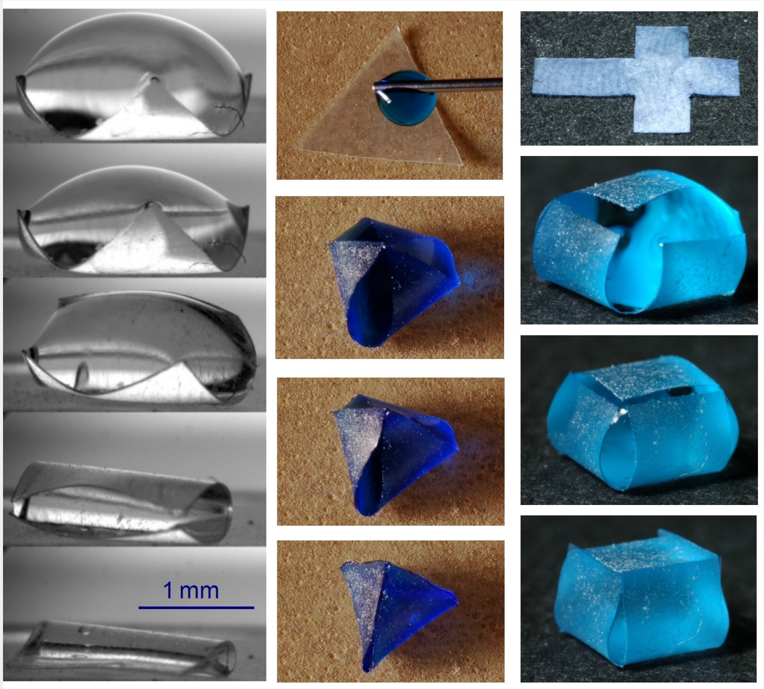


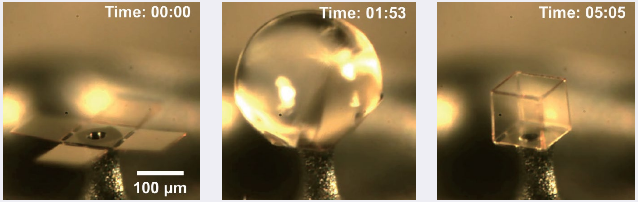
Fig. 8. Metabolic rates of a mallard oiled in a small spot; (N) normal mallards.

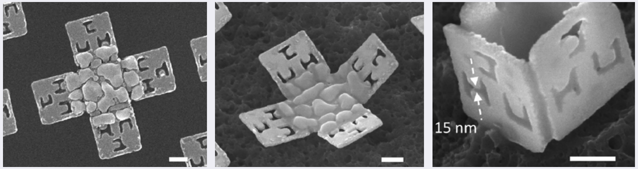
4. Elastocapillarity

Py et al. Phys. Rev. Lett. (2007)



Legrain, Tas et al. (2013)

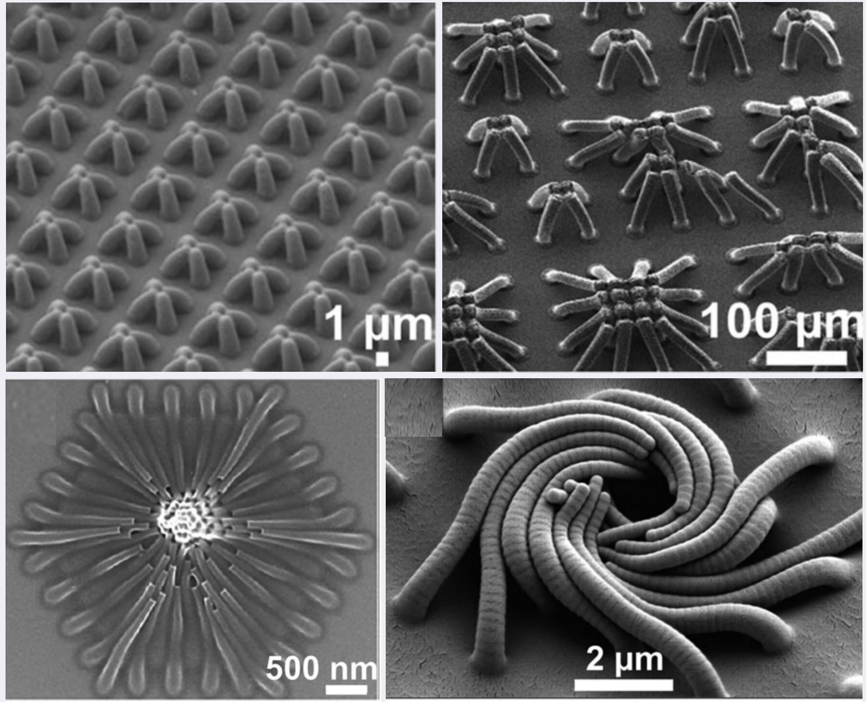




Cho & Gracias Nano Lett. (2009)

Application to self-assembly of carbon nanotubes

Tokyofashion.com



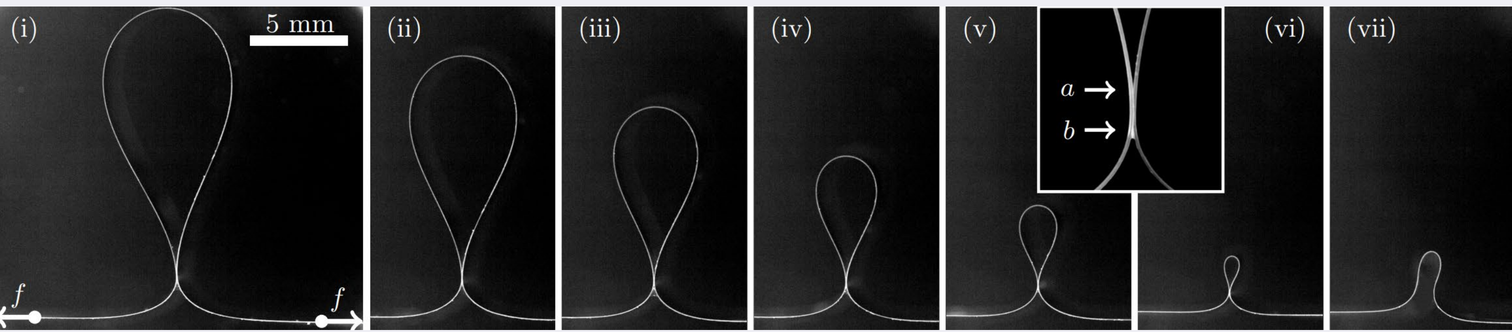
M. D. Volder and A. J. Hart. Angew. Chem. (2013)

Elastocapillarity microstructures

Li, Deng et al. Nature (2021)

5. Peeling and tearing

Related course lecture: Peeling and tearing

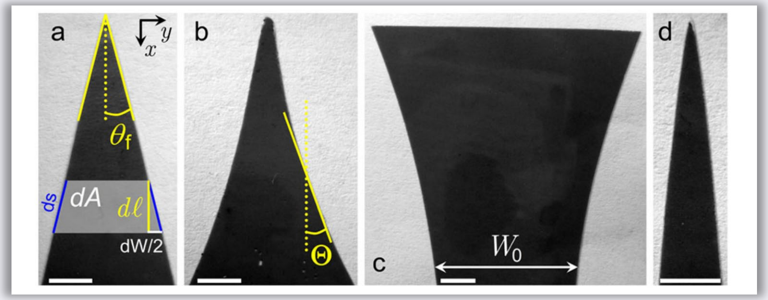


Jacques Villeglé's Arrachages I-V

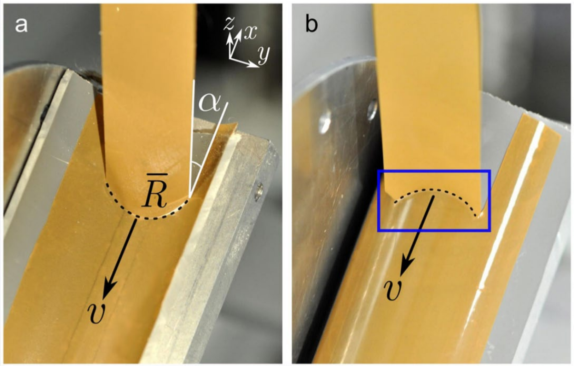
Tearing as a test for mechanical characterization of thin adhesive films

Hamm et al. Nat. Mater. (2008)

$\kappa = 0$ $\kappa < 0$ $\kappa > 0 \ \& \ W_0 > W_c$



$\kappa > 0 \ \& \ W_0 < W_c$

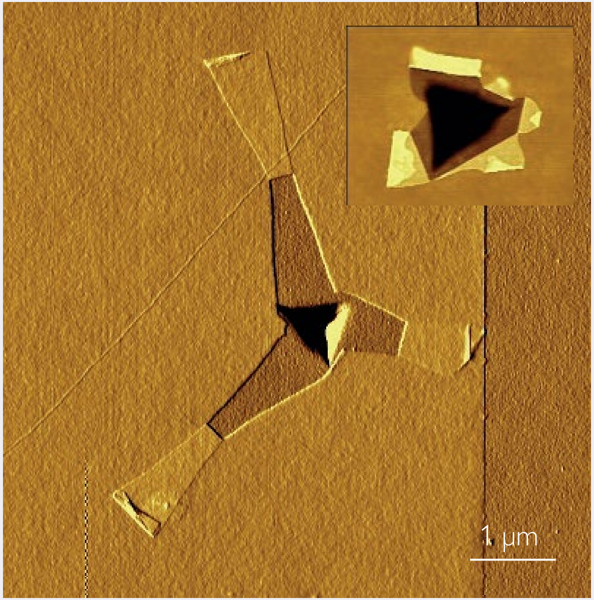


$\kappa > 0$ $\kappa < 0$

Kruglova et al. Phys. Rev. Lett. (2011)

From sub nanometers to meters

Self-assembly of graphene ribbons by self-tearing/peeling from a substrate

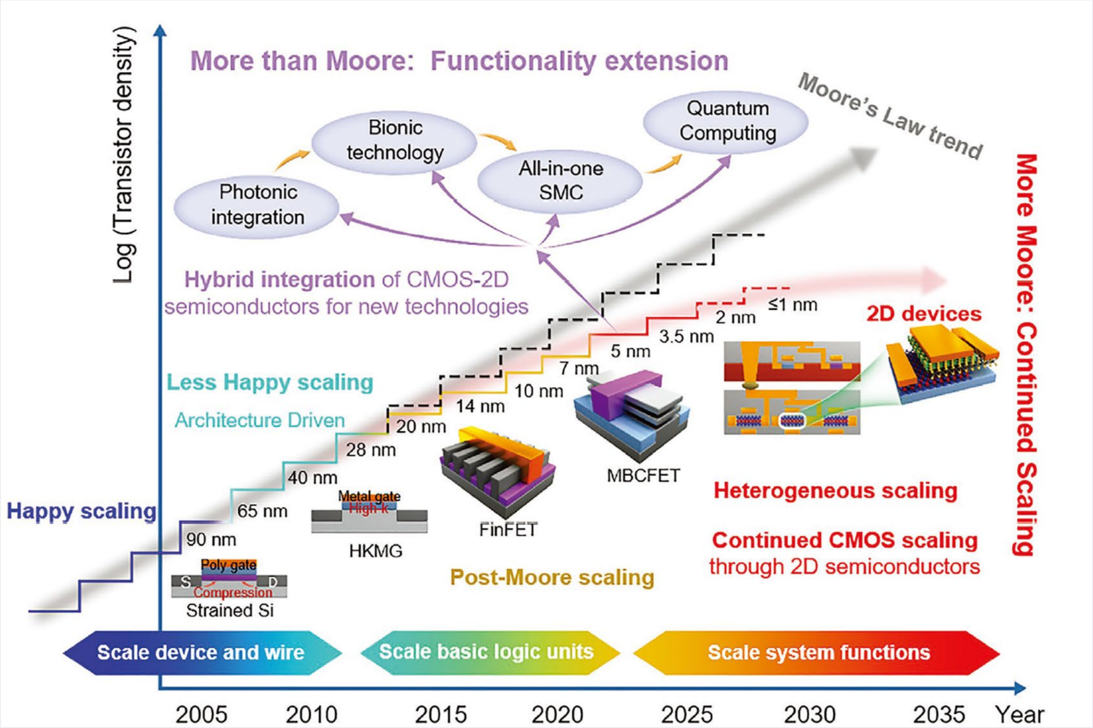


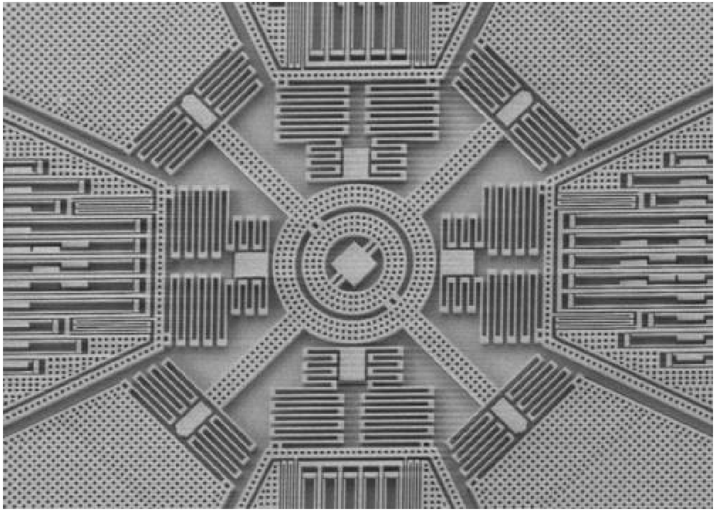
Annett and Cross Nature (2016)

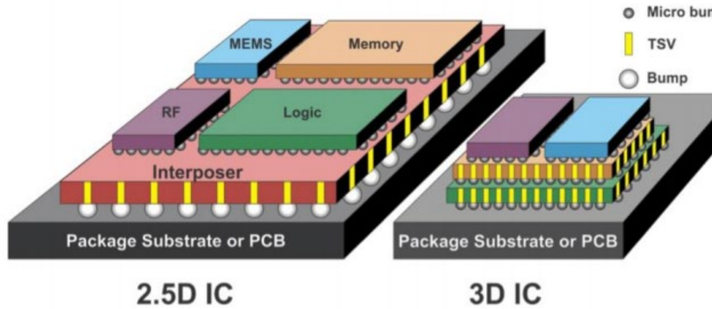
Boeing 737 of Aloha Airlines Flight 243 landed safely despite losing a part of its roof (1988)

6. Semiconductors

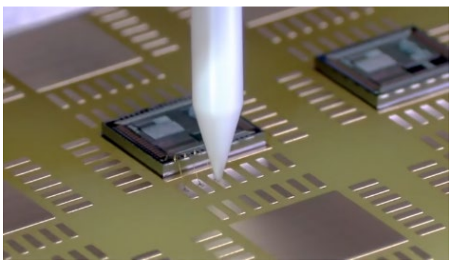
Number of components in electronic circuits has doubled every two years since the 1960s

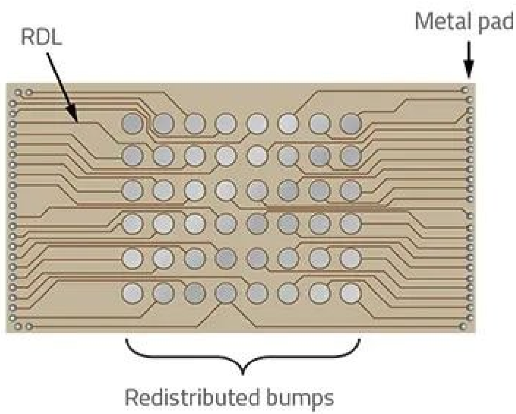





Micro bump
TSV
Bump
MEMS
Memory
RF
Logic
Interposer
Package Substrate or PCB
Package Substrate or PCB
2.5D IC
3D IC

开裂、分层、脱粘、翘曲…




RDL
Metal pad
Redistributed bumps


功率半导体封装技术
集成电路系统级封装
硅通孔三维封装技术
集成电路先进封装材料
集成电路
先进封装材料
Advanced Packaging Materials

Nowadays mechanics is an organic part of applied mathematics. An analogy of mechanics with the phoenix comes to mind. This legendary bird has appeared with practically identical magical features in the ancient legends of many cultures: Egyptian, Chinese, Hebrew, Greek, Roman, native North American, Russian. ...Death could never touch it... However, from time to time. when ii was weakened, the phoenix would carefully prepare a fire from aromatic herbs collected from throughout the world and burn itself. Everything superfluous is burnt in the fire and a new beautiful creative life opens to the phoenix.

So, what is the analogy? Mechanics now is living through a critical period. The community at large, particularly the scientific community, often considers mechan.ics to be a subject of secondary value...I am sure that a phoenix-type rebirth of mechanics is unavoidable... The reason for my confidence is the existence (not always generally recognized by society, nor political leaders) of fundamental problems of vital importance for humankind that cannot be solved without the leading participation of mechanics and applied mathematics as a whole. To mention a few of these: The prediction of earthquake, the creation of a new branch of engineering based on nano-technology; earth's non-renewable resources... (AI, Semiconductors, Aerospace...) The surge of interest in mechanics during World War II and its aftermath demonstrated this clearly.

Flow, deformation and fracture by G. I. Barenblatt, 2014, Cambridge University Press

Outline

I. About this course

II. Why I wanted to develop this course

III. Why fracture mechanics

- ❑ History of fracture mechanics
- ❑ Some scenarios in nature and engineering systems

IV. Back to the course

The methodology of this course

- “Calculus must become a pump and not a filter for the STEM pipeline.” Robert White, president of the National Academy of Engineering
  - From lecture-based practices in which students are passive learners to problem-driven in which students are active thinkers (one presentation).
  - The observation-method-new knowledge process.
  - Not a theoretician or experimentalist but problem/goal-classified “mechanician”.
- “A combination of mathematical foundations and physical pictures.”
  - Model problems of real-world systems (simplified NC or NS equations).
  - Analytical or asymptotic solutions rather than numerics.
  - Geared toward engineering while with some mathematical skills.

Goals

Goals: By means of course lectures, homework problem sets, and a project, I hope to achieve the following specific goals:

- Understanding of fundamental concepts in linear fracture mechanics
- Being capable of analyzing fracture mechanics problems in model systems
- Developing some analytical/numerical skills at modeling fracture mechanics

Not only new knowledge but also the process by which it is created and the way it can be used in natural and engineering systems!

Course Outline

- ❑ T1: Ideal strength and Griffith theory
- ❑ T2: Energy release rate
- ❑ T3: Stress intensity factors
- ❑ T4: Westergaard's stress functions
- ❑ T5: Weight function method
- ❑ T6: Cyclic loading, R-curve, Mixed mode loading
- ❑ T7: Dugdale-Barenblatt model
- ❑ T8: J integral
- ❑ T9: Peeling, tearing, buckle delamination
- ❑ T10: Adhesion theories
- ❑ T11: Dynamic fracture mechanics
- ❑ T12: Interfacial fracture mechanics
- ❑ T13: Anisotropic crack tip field
- ❑ T14: Atomic origin of surface energies

Thanks!