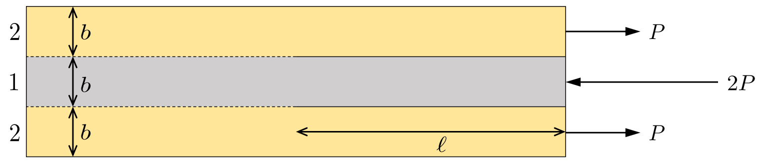
Problem 1: Double Cantilever Beam Test. Determine the energy release rate $\mathcal{G}$ for a double cantilever beam specimens loaded on the ends by applied moments M on each arm. When computing the potential energy of this system, be sure to include the work done by the applied moments. (Do you recall the work-conjugate generalized displacement for a moment?) Determine whether crack growth is stable, unstable, or neutrally stable for this specimen geometry and loading. Given that the measurement of the location of the crack tip is usually inaccurate in brittle materials, why would this type of specimen be particularly useful for determining the critical energy release rate?

Problem 2: Debonding at the interface in composites. An important problem in the design of composites is debonding by cracking along the interface between two materials. One criterion that has been proposed for brittle crack propagation along an interface is that the energy release rate $\mathcal{G}$ maintains a critical level of $\mathcal{G}_c$. Determine $\mathcal{G}$ for the push-out type specimen shown below. Material 1 has material properties E_1 and ν_1 and material 2 has properties E_2 and ν_2. Assume plane-strain conditions and that the cracks propagate in tandem along the interfaces. Regard each arm as being in plane strain tension or compression. Interpret P as a force, not a force per unit thickness.



Problem 3: Interface fracture in film-substrate systems. An elastic layer of thickness h and elastic properties μ_1 and ν_1 is bonded at a high temperature to a very large elastic quarter-space with properties of μ_2 and ν_2. Because of the thermal expansion mismatch, upon cool-down the layer is under a residual biaxial tensile stress of magnitude σ. This stress is uniform within the layer, at least at locations that are not too close to any edges. Suppose that the layer starts to debond in a 2D mode so that a crack length of a, where a is much longer than h, develops along the interface. Show that the energy release rate is

$$\mathcal{G} = \frac{(1-\nu_1)\sigma^2 h}{4\mu_1}.$$

If you do not get this answer, have you thought carefully about what the final stress state in the layer is?